



(Updated December 2018)

St Mary's Catholic Primary School



How we teach calculations: Calculation Policy for Mathematics

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About our Calculation Policy

The following calculation policy has been devised to meet requirements of the National Curriculum 2014 for the teaching and learning of mathematics, and is also designed to give pupils a consistent and smooth progression of learning in calculations across the school. Please note that early learning in number and calculation in Reception follows the 'Development Matters' EYFS document, and this calculation policy is designed to build on progressively from the content and methods established in the Early Years Foundation Stage.

Age stage expectations

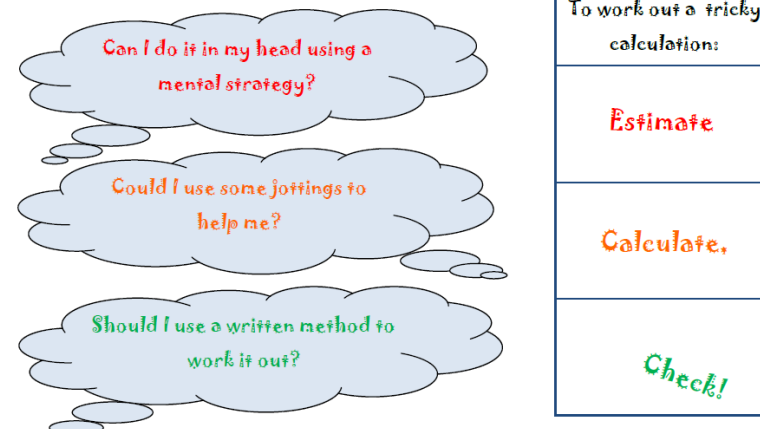
The calculation policy is organised according to age stage expectations as set out in the National Curriculum 2014, **however it is vital that pupils are taught according to the stage that they are currently working at**, being moved onto the next level as soon as they are ready, or working at a lower stage until they are secure enough to move on.

Providing a context for calculation:

It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

Choosing a calculation method:

Children need to be taught and encouraged to use the following processes in deciding what approach they will take to a calculation, to ensure they select the most appropriate method for the numbers involved:



EYFS and Year 1 – Subitising

Subitising: a skill we all use but are unlikely to remember learning. Now ‘subitising to 5’ is explicitly specified in the pilot Early Learning Goals (ELG) for Mathematics.

So, what is subitising? Why is it important? And how do practitioners provide opportunities to develop this skill in young children?

The pilot [Framework for Early Years Foundation Stage](#) has been published and is due to be piloted by 25 schools in 2018/19. Within this framework sit the proposed *Early Learning Goals* (p12/13), including those for mathematics. There are two goals for mathematics: Number, and Numerical Patterns. Within

Number, the second of three bullet points is: Subitise (recognise quantities without counting) up to 5.

What is Subitising?

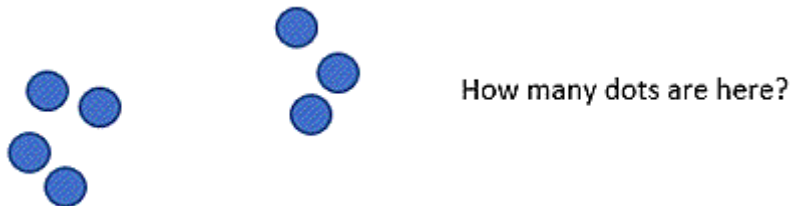
Sarama and Clements (2009)¹ defined subitising as “A quick attention toward numerosity when viewing a small set of objects”.

It is the ability to quickly recognising how many objects are in a group without actually counting them. As adults, most people can subitise up to five objects – this is called perceptual subitising. We also subitise larger numbers of objects by ‘seeing’ them in groups of five or less and combining these – this is called conceptual subitising.

Why is it important?

Our ability to perceive the exact quantity of small groups of numbers, and to put these numbers together to perceive the quantity of larger groups, is fundamental to our understanding of how numbers partition.

For example:



...you have probably recognised 4 and 3 and know that they add to make 7, most likely without any counting or calculation. If this is the case, you have subitised. This is an important part of developing number sense. Subitising this group of 7 is far more efficient than either using a touch-counting method, or perceiving 4, then counting on.

NCETM Assistant Director for Early Years and Primary, Viv Lloyd, says, “Subitising is so critical because you are starting to see the numbers within numbers, so once you start subitising to 6, you are starting to see 5 and 1, 4 and 2, or 3 and 3, and that is building a sense of the 6-ness of six as well as being

introduced to the number bonds. Children can playfully experience this and draw on that knowledge in later years to recall those facts. Separation and recombining is a more effective calculation strategy than 'counting on' or 'counting back'. So counting on and counting back is not in the pilot Early Learning Goals (whereas it was previously in the old ones), and subitising is now explicitly specified."

See: <https://www.ncetm.org.uk/resources/52560>

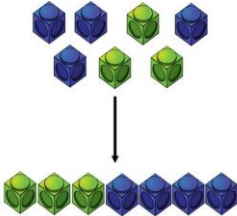
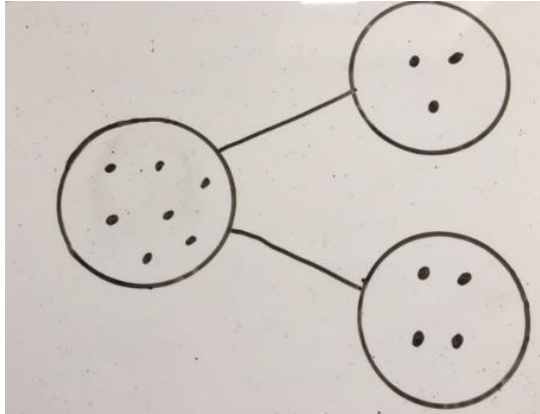
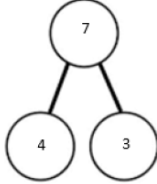
What activities could we do to encourage children to subitise?

We need to provide opportunities for children to develop this skill.

- 'Accidentally' spilling some counters / teddies / dinosaurs on the floor. How many are there? How do you know? How did you see it? Did you see it another way?
- Games that involve hiding a small number of objects in a box or under a cloth, and getting children to take a peek and say how many there are.
- Throwing a number (up to 5) of two-sided beanbags. Children then say what they can see "I can see 2 patterned and 1 plain beanbag – there are 3 beanbags altogether". A more complex version of this would be to hide some of a known number of beanbags. "I have 3 beanbags. I can see 2, so there must be 1 in the box."
- Using 5 seeds, plant them in 2 flowerpots, talking about how many seeds are planted in each pot and making a total, for example, "2 seeds are planted in my pot and 3 seeds are planted in your pot. There are 5 seeds altogether".

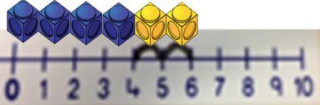

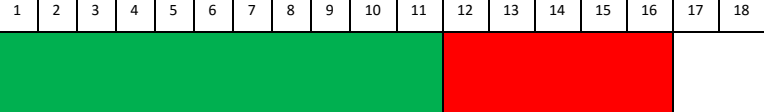
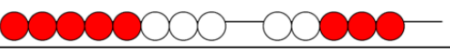
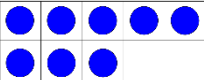
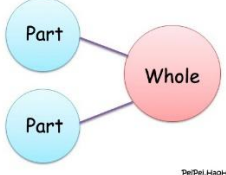
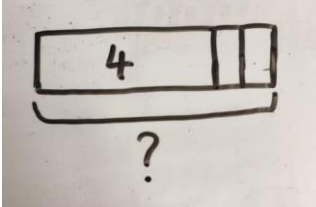


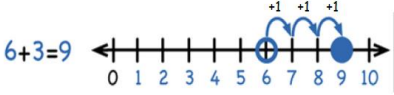

Reception Combine two parts to make a whole.

Addition - Reception

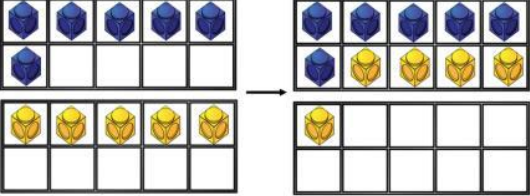
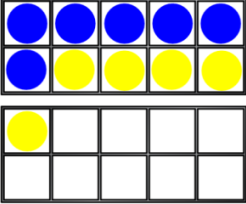
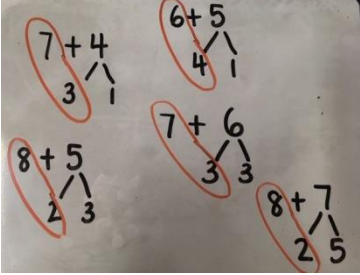
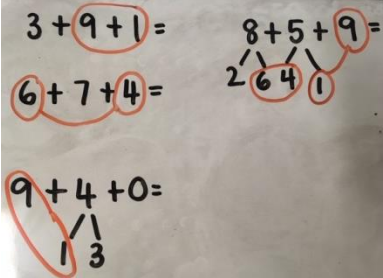
Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).</p> 	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p> 	<p>$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven.</p> 

Year 1 Add with numbers up to 20

Addition – Year 1

Concrete	Pictorial	Abstract
<p>Children should: Have access to a wide range of counting equipment, everyday objects, number tracks and number lines (ensure number lines are varied e.g. horizontal, vertical, circular etc.) hundred square and be shown numbers in different contexts.</p> <p>Counting on using number lines using cubes or Addacus bars.</p> <p>Cubes</p>  <p>Eg. Give each child 10 cubes. Make into a tower and ask children to put it under the desk. Ask children to show you 8. Question: How many have you got left under the table? Why? Develops fluency.</p>  <p>Addacus</p> <p>Use the different coloured Addacus bars so that the different amounts can be seen. EG. $11 + 5 =$</p>  <p>$11 + 5 = 16$</p> <p>This builds on from prior learning of adding by combining two sets of objects into one group (5 cubes and 3 cubes) in Early Years.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.</p> <p>$8 + 5$</p>  </div> <p>Ten frame / Double ten frame</p>  <p>Perhaps use double-sided counters See https://nrich.maths.org/10742 for more ten frame resources.</p>	<p>Use the Part-Part-Whole model</p>  <p>This then leads to...</p> <p>A bar model which encourages the children to count on, rather than count all.</p>  <p>Children to draw the ten frame in their book</p>  	<p>Use numbered number lines to add, by counting on in ones. Explore efficient methods - encourage children to start with the larger number and count on.</p>  <p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$</p>  <p>Read and write the addition (+) and equals (=) signs within number sentences.</p> <p>Draw out the RELATIONSHIPS: Use the ten frame / double ten frame to support this. EG. $1 + 9 = 10$ $2 + 8 = 10$ What do you notice? What happens if I do this...? $_ + 6 = 10$</p> <p>Or $_ + 8 = 11$ How do you know?</p>

Addition – Year 1

Concrete	Pictorial	Abstract
<p>Regrouping to make 10; using ten frames and counters/cubes or using Addacus.</p> <p>$6 + 5$</p> 	<p>Children to draw the ten frame and counters/cubes.</p> 	<p>Children to use the concept of regrouping to help them:</p>  <p>Children can apply this to more complex problems:</p>  <p>Interpret addition number sentences and solve missing box problems to gain an understanding of equality, using concrete objects and number line addition to solve them:</p> <p> $8 + 3 = \square$ $15 + 4 = \square$ $5 + 3 + 1 = \square$ $\square + \square = 6$ </p> <p> $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$ </p>

Key vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line

addition sign is equivalent to

$$9 + 5 = 14$$

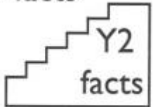
addend addend sum

Addition (commutative)

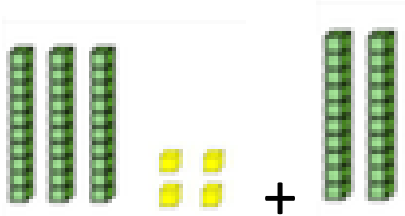
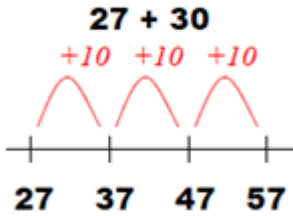
Addition – Year 1

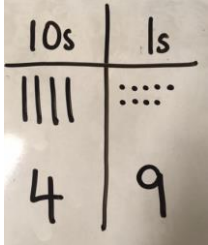
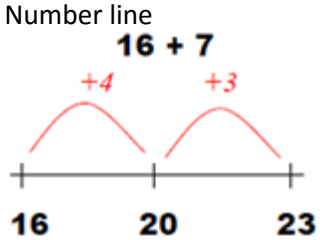
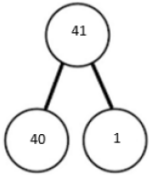
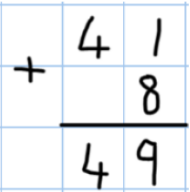
Key skills for addition at Y1:

- Read and write numbers to 100 in numerals, incl. 1–20 in words
- Recall bonds to 10 and 20, and addition facts within 20
- Count to and across 100
- Count in multiples of 1, 2, 5 and 10
- Solve simple 1-step problems involving addition, using objects, number lines and pictorial representations.

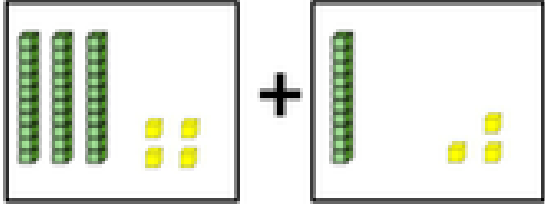
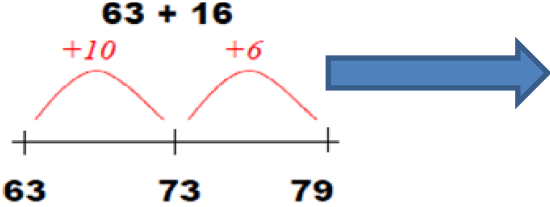
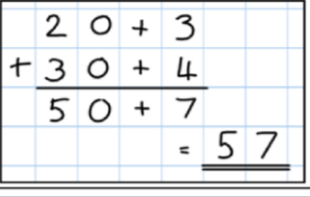
		Adding 1		Bonds to 10		Adding 10		Bridging/ compensating		Y1 facts		
		Adding 2		Adding 0		Doubles		Near doubles				
+		0	1	2	3	4	5	6	7	8	9	10
0		0+0	0+1	0+2	0+3	0+4	0+5	0+6	0+7	0+8	0+9	0+10
1		1+0	1+1	1+2	1+3	1+4	1+5	1+6	1+7	1+8	1+9	1+10
2		2+0	2+1	2+2	2+3	2+4	2+5	2+6	2+7	2+8	2+9	2+10
3		3+0	3+1	3+2	3+3	3+4	3+5	3+6	3+7	3+8	3+9	3+10
4		4+0	4+1	4+2	4+3	4+4	4+5	4+6	4+7	4+8	4+9	4+10
5		5+0	5+1	5+2	5+3	5+4	5+5	5+6	5+7	5+8	5+9	5+10
6		6+0	6+1	6+2	6+3	6+4	6+5	6+6	6+7	6+8	6+9	6+10
7		7+0	7+1	7+2	7+3	7+4	7+5	7+6	7+7	7+8	7+9	7+10
8		8+0	8+1	8+2	8+3	8+4	8+5	8+6	8+7	8+8	8+9	8+10
9		9+0	9+1	9+2	9+3	9+4	9+5	9+6	9+7	9+8	9+9	9+10
10		10+0	10+1	10+2	10+3	10+4	10+5	10+6	10+7	10+8	10+9	10+10

Year 2 Add with 2-digit numbers. *Develop mental fluency with addition and place value involving 2-digit numbers, then establish more formal methods.*

Add 2-digit numbers and tens	Concrete	Pictorial	Abstract											
	<p>Use dienes</p> <p>Eg. $34 + 20$</p> 	<p>$27 + 30$</p> <p>$+10 +10 +10$</p>  <p>$27 \quad 37 \quad 47 \quad 57$</p> <p>Use bar modelling alongside this too.</p> <table border="1" data-bbox="952 845 1478 1037"> <tr> <td>27</td> <td>10</td> <td>10</td> <td>10</td> </tr> <tr> <td>27</td> <td colspan="3">30</td> </tr> <tr> <td colspan="4">57</td> </tr> </table>	27	10	10	10	27	30			57			
27	10	10	10											
27	30													
57														

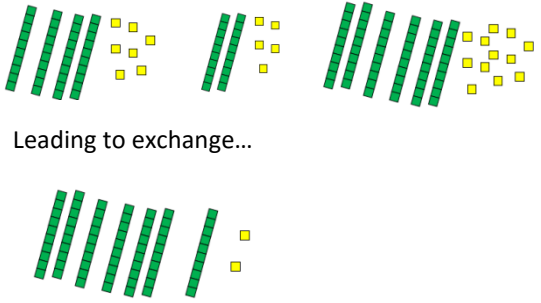
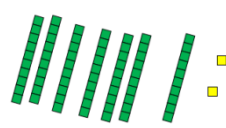
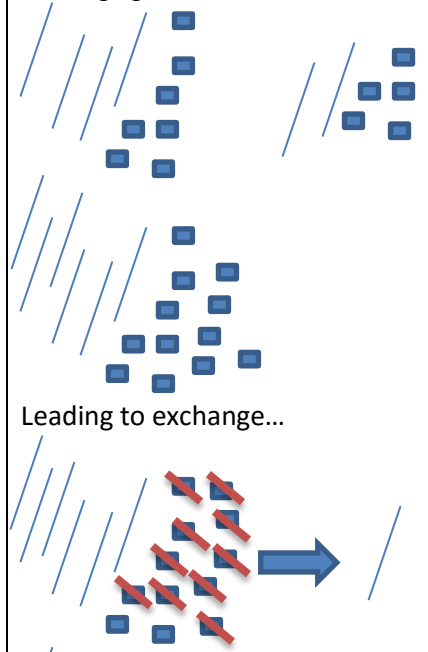
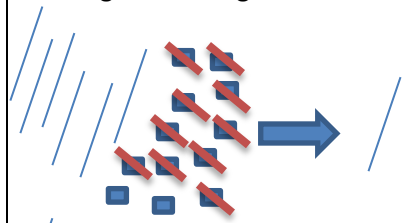
Concrete	Pictorial	Abstract								
<p>TU + U using base 10. Continue to develop understanding of partitioning and place value. 41 + 8</p>	<p>Use empty number lines, concrete equipment, hundred squares etc. to build confidence and fluency in mental addition skills.</p> <p>Children to represent the base 10 e.g. lines for tens and dot for ones. Encourage children to draw the dots in the 'ten frame' pattern to help them to subitise.</p>  <p>Number line</p>  <p>Bar modelling</p> <table border="1" data-bbox="945 1168 1482 1439"> <tr> <td>16</td> <td>7</td> </tr> <tr> <td>16</td> <td>4</td> <td>3</td> </tr> <tr> <td>20</td> <td>3</td> </tr> <tr> <td>23</td> </tr> </table>	16	7	16	4	3	20	3	23	<p>41 + 8</p>  <p>1 + 8 = 9 40 + 9 = 49</p> 
16	7									
16	4	3								
20	3									
23										

Addition – Year 2

STEP 1: Only provide examples that <u>do NOT</u> cross the tens boundary until they are secure with the method itself.										
	Concrete	Pictorial	Abstract							
Add pairs of 2-digit numbers, moving to the partitioned column method when secure adding tens and units:	Hundred square Dienes base 10 apparatus $34 + 13$ 	Number line $63 + 16$ 	Partitioned column method $23 + 34:$ 							
		Bar modelling <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 70%; text-align: center;">63</td> <td style="width: 30%; text-align: center;">16</td> </tr> <tr> <td style="text-align: center;">63</td> <td style="text-align: center;">10 6</td> </tr> <tr> <td style="text-align: center;">63</td> <td style="text-align: center;">16</td> </tr> <tr> <td colspan="2" style="text-align: center; border: 1px solid black;">79</td> </tr> </table>	63	16	63	10 6	63	16	79	
63	16									
63	10 6									
63	16									
79										

Addition – Year 2

STEP 2: Once children can add a multiple of ten to a 2-digit number mentally (e.g. 80+11), they are ready for adding pairs of 2-digit numbers that DO cross the tens boundary (e.g. 58 + 43).

<p>Add pairs of 2-digit numbers, moving to the partitioned column method when secure adding tens and units:</p>	<p>Concrete</p> <p>To support understanding, pupils may physically make and carry out the calculation with Dienes Base 10 apparatus or place value counters, then compare their practical version to the written form, to help them to build an understanding of it.</p> <p>47 + 25</p>  <p>Leading to exchange...</p> 	<p>Pictorial</p> <p>Children to draw their rods and dots. Cross out using a different colour when exchanging.</p>  <p>Leading to exchange...</p>  <p>Bar modelling</p> <p>47 + 25</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="width: 50px;">47</td> <td style="width: 50px;">25</td> <td colspan="2"></td> </tr> <tr> <td>40</td> <td>20</td> <td>7</td> <td>5</td> </tr> <tr> <td colspan="2">60</td> <td colspan="2">12</td> </tr> <tr> <td colspan="4">72</td> </tr> </table>	47	25			40	20	7	5	60		12		72				<p>Abstract</p> $ \begin{array}{r} 40 + 7 \\ + 20 + 5 \\ \hline 60 + 12 = 72 \end{array} $ <p>58 + 43:</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>5</td><td>0</td><td>+</td><td>8</td><td></td><td></td> </tr> <tr> <td>4</td><td>0</td><td>+</td><td>3</td><td></td><td></td> </tr> <tr> <td colspan="6"><hr/></td> </tr> <tr> <td>9</td><td>0</td><td>+</td><td>11</td><td></td><td></td> </tr> <tr> <td colspan="6"><hr/></td> </tr> <tr> <td></td><td></td><td></td><td></td><td>=</td><td>101</td> </tr> </table> <div style="border: 2px solid black; border-radius: 50%; padding: 20px; width: fit-content; margin: 20px auto;"> <p>STEP 3: Children who are confident and accurate with this stage should move onto the expanded addition methods with 2 and 3-digit numbers (see Y3).</p> </div>	5	0	+	8			4	0	+	3			<hr/>						9	0	+	11			<hr/>										=	101
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Key vocabulary:

add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, addition, column, tens boundary

addition sign is equivalent to

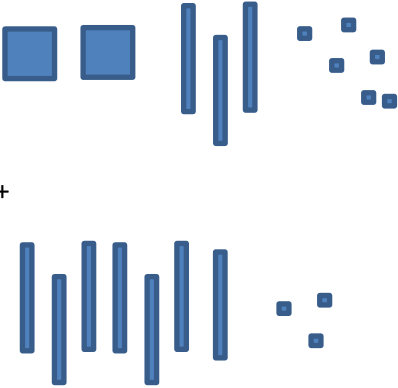
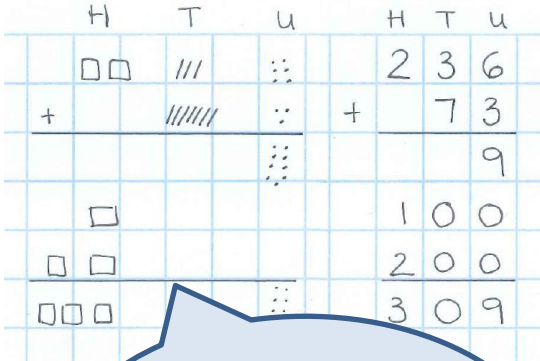
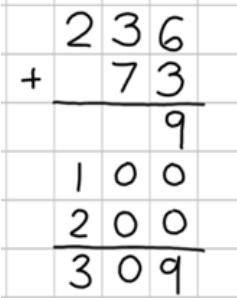
$$9 + 5 = 14$$

addend addend sum

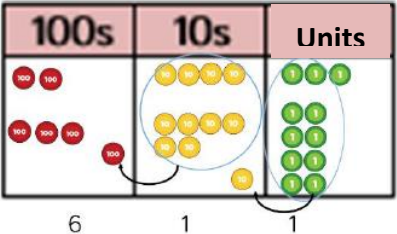
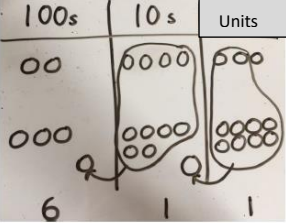
Addition (commutative)**Key skills for addition at Y2:**

- Add a 2-digit number and ones (e.g. 27 + 6)
- Add a 2-digit number and tens (e.g. 23 + 40)
- Add pairs of 2-digit numbers (e.g. 35 + 47)
- Add three single-digit numbers (e.g. 5 + 9 + 7)
- Show that adding can be done in any order (the commutative law).
- Recall bonds to 20 and bonds of tens to 100 (30 + 70 etc.)
- Count in steps of 2, 3 and 5 and count in tens from any number.
- Understand the place value of 2-digit numbers (tens and units)
- Compare and order numbers to 100 using < > and = signs.
- Read and write numbers to at least 100 in numerals and words.
- Solve problems with addition, using concrete objects, pictorial representations, involving numbers, quantities and measures, and applying mental and written methods.

Year 3 Add numbers with up to three digits.

Step 1 Introduce the expanded column addition method:						
Concrete	Pictorial	Abstract				
<p>Use dienes to support understanding.</p> <p>236 + 73</p> 	<p>Children to draw rods and dots.</p> <p>Then move onto teaching this alongside the expanded column method.</p>  <div style="border: 1px solid black; border-radius: 50%; padding: 10px; margin: 10px auto; width: 80%; background-color: #e0f0ff;"> <p>Ensure that children put their picture (rod / dot etc.) in the correct column. This will support understanding when children move onto carrying (step 2)</p> </div> <p>Continue to use the bar model to represent the problems.</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td colspan="2" style="text-align: center;">309</td> </tr> <tr> <td style="text-align: center;">236</td> <td style="text-align: center;">79</td> </tr> </table>	309		236	79	<p>NB: Add the units first, in preparation for the compact method.</p>  <p>In order to carry out this method of addition:</p> <ul style="list-style-type: none"> • Children need to recognise the value of the hundreds, tens and units without recording the partitioning. • Pupils need to be able to add in columns.
309						
236	79					

Step 2: Move to the compact **column addition** method, with “carrying”.

Concrete	Pictorial	Abstract
<p>Use dienes (as above – step 1)</p> <p>Move onto Place Value counters when children have a secure conceptual understanding.</p> 		<p>Children who are very secure and confident with 3-digit expanded column addition should be moved onto the compact column addition method, being introduced to “carrying” for the first time. Compare the expanded method to the compact column method to develop an understanding of the process and the reduced number of steps involved.</p> <div style="display: flex; align-items: center;"> <div style="margin-right: 20px;"> <p>Add units first.</p> <p>'Carry' numbers underneath the bottom line.</p> </div> <div style="margin-right: 20px;"> $\begin{array}{r} 236 \\ + 73 \\ \hline 309 \\ 1 \end{array}$ </div> <div style="border: 1px solid black; border-radius: 50%; padding: 10px; width: fit-content;"> <p>Remind pupils the actual value is three tens add seven tens, not three add seven, which equals ten tens</p> </div> </div>

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, **hundreds boundary, increase, vertical, "carry", expanded, compact**

addition sign is equivalent to

$$9 + 5 = 14$$

addend addend sum

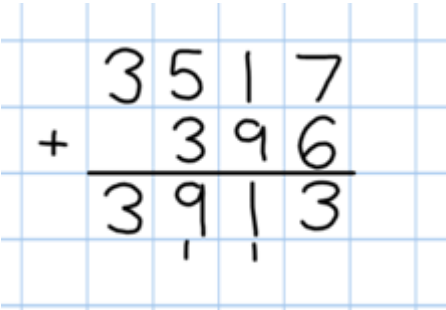
Addition (commutative)

Key skills for addition at Y3:

- Read and write numbers to 1000 in numerals and words.
- Add 2-digit numbers mentally, incl. those exceeding 100.
- **Add a three-digit number and ones mentally (175 + 8)**
- **Add a three-digit number and tens mentally (249 + 50)**
- **Add a three-digit number and hundreds mentally (381 + 400)**
- Estimate answers to calculations, using inverse to check answers.
- Solve problems, including missing number problems, using number facts, place value, and more complex addition.
- Recognise place value of each digit in 3-digit numbers (hundreds, tens, ones.)
- Continue to practise a wide range of mental addition strategies, ie. number bonds, adding the nearest multiple of 10, 100, 100 and adjusting, using near doubles, partitioning and recombining.

Video clip: [Demonstration of expanded 3-digit column addition](#)

Year 4 Add numbers with up to four digits.

Concrete	Pictorial	Abstract
Use dienes or PV counters	Children to draw the PV grid in their book with the PV counters.	<p>Move from expanded addition to the compact column method, adding units first, and “carrying” numbers underneath the calculation. Also include money and measures contexts.</p> <p>Introduce the compact column addition method by asking children to add the two given numbers together using the method that they are familiar with (expanded column addition—see Y3). Teacher models the compact method with carrying, asking children to discuss similarities and differences and establish how it is carried out.</p> <p>e.g. $3517 + 396 = 3913$</p>  <ul style="list-style-type: none"> • Add units first. • “Carry” numbers underneath the bottom line and cross them off once counted. • Reinforce correct place value by reminding them the actual value is 5 hundreds add 3 hundreds, not 5 add 3, for example.

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, vertical, „carry“, expanded, compact, **thousands, hundreds, digits, inverse**

addition sign is equivalent to

$$9 + 5 = 14$$

addend addend sum

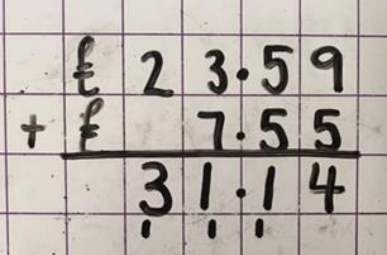
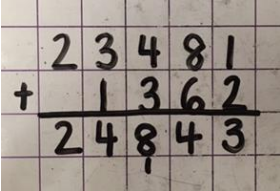

Addition (commutative)

Key skills for addition at Y4:

- Select most appropriate method: mental, jottings or written and explain why.
- Recognise the place value of each digit in a four-digit number.
- Round any number to the nearest 10, 100 or 1000.
- Estimate and use inverse operations to check answers.
- Solve 2-step problems in context, deciding which operations and methods to use and why.
- Find 1000 more or less than a given number.
- Continue to practise a wide range of mental addition strategies, ie. number bonds, add the nearest multiple of 10, 100, 1000 and adjust, use near doubles, partitioning and recombining.
- Add numbers with up to 4 digits using the formal written method of column addition
- Solve 2-step problems in contexts, deciding which operations and methods to use and why.
- Estimate and use inverse operations to check answers to a calculation.

Year 5 Add numbers with more than 4 digits including money, measures and decimals with different numbers of decimal places.

Addition – Year 5

Concrete	Pictorial	Abstract
<p>Use PV counters.</p>	<p>Children to draw the PV grid in their book with the PV counters.</p>	<p>The decimal point should be aligned in the same way as the other place value columns, and must be in the same column in the answer.</p>  <p>Numbers should exceed 4 digits.</p>  <p>Pupils should be able to add more than two values, carefully aligning place value columns.</p>  <div data-bbox="1155 1209 1462 1461" style="border: 1px solid blue; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> <p>Say "6 tenths add 7 tenths" to reinforce place value.</p> </div> <div data-bbox="1771 1129 2175 1565" style="border: 1px solid blue; border-radius: 50%; padding: 10px; width: fit-content; margin: 10px auto;"> <p>Empty decimal places can be filled with zero to show the place value in each column</p> </div>

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, "carry", expanded, compact, vertical, thousands, hundreds, digits, inverse & decimal places, decimal point, tenths, hundredths, thousandths

addition sign is equivalent to

$$9 + 5 = 14$$

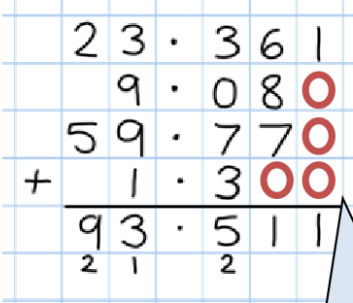
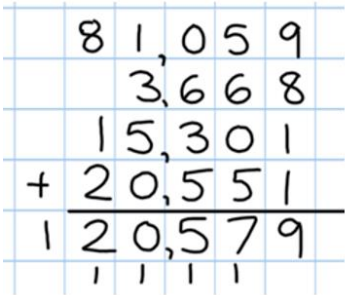
addend addend sum

Addition (commutative)

Key skills for addition at Y5:

- Add numbers mentally with increasingly large numbers, using and practising a range of mental strategies ie. add the nearest multiple of 10, 100, 100 and adjust; use near doubles, inverse, partitioning and re-combining; using number bonds.
- Use rounding to check answers and accuracy.
- Solve multi-step problems in contexts, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000.
- Add numbers with more than 4 digits using formal written method of columnar addition.

Year 6 Add several numbers of increasing complexity

Concrete	Pictorial	Abstract
<p>Use PV counters.</p>	<p>Children to draw the PV grid in their book with the PV counters.</p>	<p>Adding several numbers with different numbers of decimal places (including money and measures):</p> <ul style="list-style-type: none"> • Tenths, hundredths and thousandths should be correctly aligned, with the decimal point lined up vertically including in the answer row. • Zeros could be added into any empty decimal places, to show there is no value to add.  <p>Adding several numbers with more than four digits:</p> 

Key vocabulary: add, more, plus, and, make, altogether, total, equal to, equals, double, most, count on, number line, sum, tens, units, partition, plus, addition, column, tens boundary, hundreds boundary, increase, "carry", expanded, compact, vertical, thousands, hundreds, digits, inverse, decimal places, decimal point, tenths, hundredths, thousandths

addition sign is equivalent to

$$9 + 5 = 14$$

addend addend sum

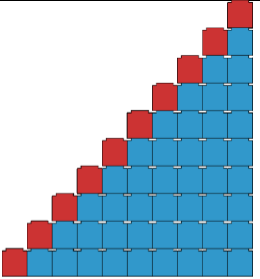
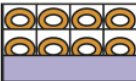

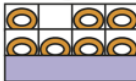
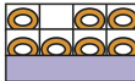
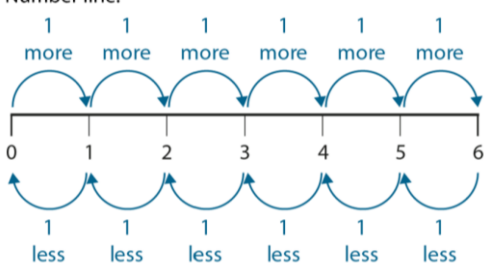

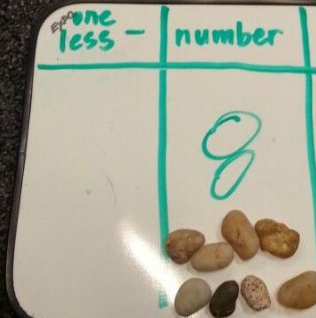
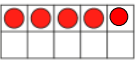
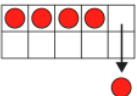
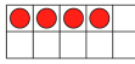
Addition (commutative)

Key skills for addition at Y6:

- Perform mental calculations, including with mixed operations and large numbers, using and practising a range of mental strategies.
- Solve multi-step problems in context, deciding which operations and methods to use and why.
- Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.
- Read, write, order and compare numbers up to 10 million and determine the value of each digit.
- Round any whole number to a required degree of accuracy. ☑ Pupils understand how to add mentally with larger numbers and calculations of increasing complexity.

Subtraction – Reception

Reception Finding one less

Concrete	Pictorial	Abstract										
 <p>Using the multi-link tower and other varied representations (such as counters or tens frames or addacus bars) start to explore one less. Ask children to describe the consecutive number pairs in full sentences. Eventually move towards the generalised statement: 'The number before a given number is one less'.</p> <p>Eg. Four is one less than five.</p> <table border="1" data-bbox="190 925 604 1093"> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td></td> </tr> </table> <p><i>'First there were eight doughnuts. Then one was eaten. Now there are seven doughnuts.'</i></p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>First</p>  <p>8</p> </div> <div style="text-align: center;"> <p>Then</p>   <p>- 1</p> </div> <div style="text-align: center;"> <p>Now</p>  <p>7</p> </div> </div> <p style="text-align: center;">$8 - 1 = 7$</p>											<p>Number line:</p>    <p><i>'First we had five. Then we subtracted one. Now we have four.'</i></p> <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;"> <p>First</p>  <p>5</p> </div> <div style="text-align: center;"> <p>Then</p>  <p>- 1</p> </div> <div style="text-align: center;"> <p>Now</p>  <p>4</p> </div> </div> <p style="text-align: center;">$5 - 1 = 4$</p>	<p>One less than 4 is ...</p> <p>First I had 4 sweets, then I ate 1. Now I have...</p>

Subtraction – Reception

Key vocabulary:

take away, less than, the difference, subtract, minus, fewer, decrease

First...then...now...

Key skills for subtraction in EYFS:

Count reliably with numbers from 1 - 20

Place numbers from 1-20 in order

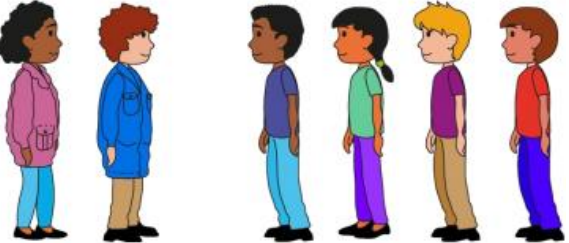
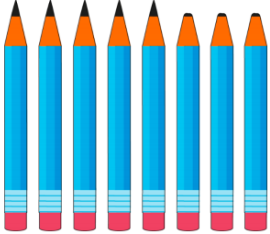

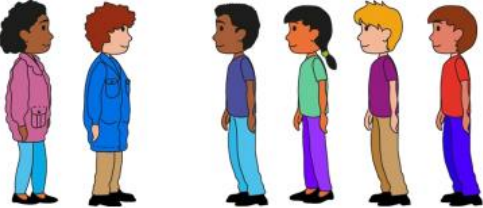
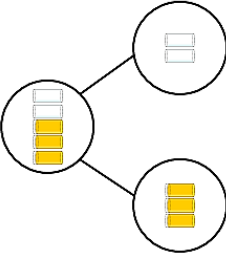
Say which number is one more or one less than a given number (1-20)

Using quantities and objects, they subtract two single-digit numbers and count back to find the answer

They solve problems with halving

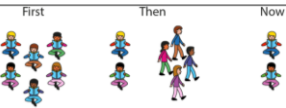
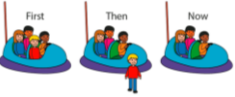
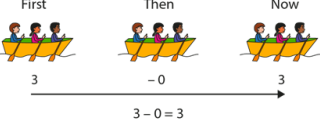
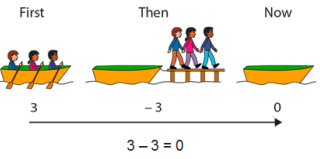
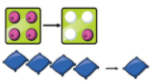

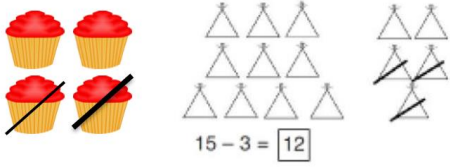
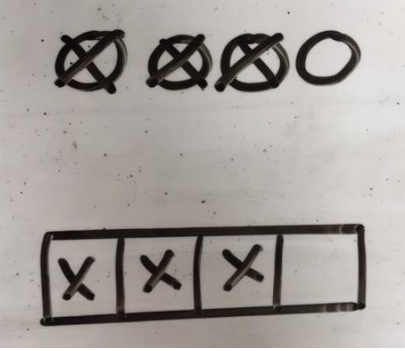

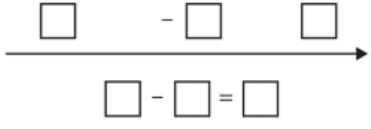
Year 1 Breaking down a whole into two or more parts (partitioning)

Having experienced finding an unknown part represented within an addition equation, children can progress to finding the unknown part using a subtraction structure. This is the partitioning structure of subtraction there are two distinct parts, one of which is unknown (it can also be thought of as the ‘not’ structure). This is different to the REDUCTION structure (see below), where one part is removed or taken away from the whole.

Concrete	Pictorial	Abstract
<p>Use concrete resources to model the partitioning structure. Present children with contextual examples like these, for numbers within ten.</p> <p>There are six children. Two have put their coats on. How many have not put their coats on?</p>  <p>There are eight pencils. Five have been sharpened. How many have not been sharpened?</p>  <p>More examples:</p> <p>There are five windows Three are open. How many are closed?</p> <p>There are seven children. Six of them are having packed lunch. How many of them are not having a packed lunch.</p>	<p>Now present these problems using pictures / children to draw them on their whiteboards / paper.</p> <p>Use pictorial prompts. Children to come up with their own problems based on the picture.</p>  <p>THEN use multi-link to represent the children / pencils / whatever is in the question.</p>	<p>There are six children. Two have put their coats on. How many have not put their coats on?</p>  <p>$6 - 4 = 2$</p> <p>The six represents all of the children. The minus 2 represents the children who have put their coats on. The 4 represents the children who have not put their coats on.</p> <p>Use part-part-whole diagrams.</p> 

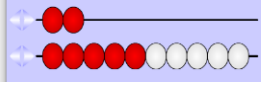
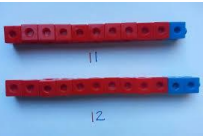
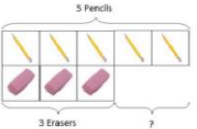
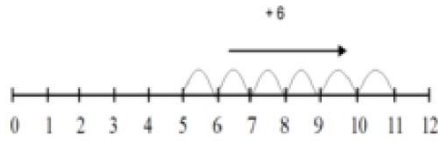
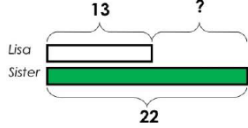
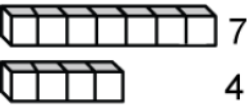
Year 1 Subtraction as reduction (taking away)

Subtraction – Year 1

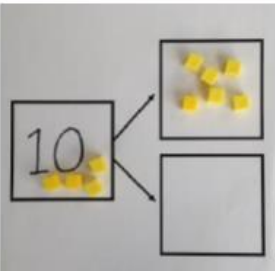
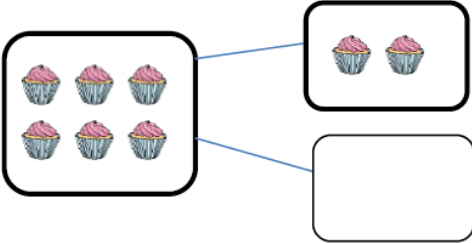
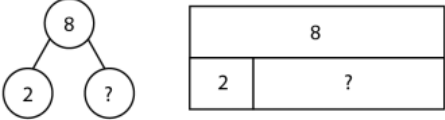
Concrete	Pictorial	Abstract
<p>Physically taking away and removing objects from a whole.</p> <p>Ten frames, addacus resources, cubes and other items such as beanbags, conkers, leaves etc. could be used as well as the children themselves.</p> <p>Use “first...then...now...”</p> <p>Children to act it out: “First there were five children in the book corner. Then two children left the book corner. Now there are three children in the book corner.”</p>  <p>When modelling, draw children’s attention to the ‘then’ part of the story being the decrease in value.</p>  <p>What does the 4 represent? What does the minus 1 represent? What does the 3 represent?</p> <p>Also consider cases where the subtrahend is 0. You will need to consider carefully how to represent the ‘0’ in the ‘then’ box. This should not be empty. The ‘0’ signifies that nothing changes.</p>   <p>Encourage children to physically remove items using touch counting.</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the items in the boxes.</p> <p>First Then Now</p>  <p>NB: IT IS IMPORTANT THAT THE CHILDREN CROSS OFF WHAT THEY ARE SUBTRACTING FROM THE ‘FIRST’ BOX IN ORDER THAT THEY DEVELOP AN UNDERSTANDING THAT THEY ARE SUBTRACTING FROM A WHOLE AMOUNT. THERE ARE NOT TWO AMOUNTS.</p> <p>Children to draw the concrete resources they are using and cross out the correct amount.</p>  <p>The bar model can also be used.</p> 	<p>Move onto the written:</p> <p>First Then Now</p>   <p>$4 - 1 = 3$</p> <p>What does the 4 represent? What does the minus 1 represent? What does the 3 represent?</p> <div style="border: 1px solid black; padding: 10px; margin-top: 20px;"> <p>Act out problems in different ways</p> <p>What is the same? What is different?</p> <p>Mary had 7 letters in her bag and she posted 3. How many did she have left? $7 - 3 = ?$</p> <p>Mary had 7 letters in her bag and after she posted some, she had 4 left. How many did she post? $7 - ? = 4$</p> <p>Mary had some letters and after posting 3, she had 4 left. How many did she start with? $? - 3 = 4$</p> </div>

Year 1 Subtraction as difference

This will be introduced practically with the language **‘find the difference/distance between’** and **‘how many more?’** in a range of familiar contexts.

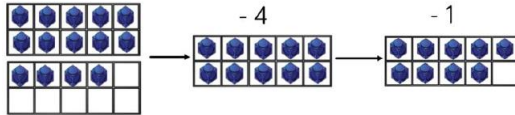
Concrete	Pictorial	Abstract
<p>Use multilink, bead strings, addacus, objects.</p>  <p>Step 1: Counting BACK Difference should be introduced through counting back from the larger number to the smaller number. This should then be shown on the number line.</p> <p>Children should understand that the number sentence $7 - 3 = ?$ is calculated by performing $7 - ? = 3$</p> <p>Step 2: Counting ON Difference can then be explored by asking: “What would the answer be if we counted on from the smaller number to the larger number?” This should then be shown on the number line. Children should understand that the number sentence $7 - 3 = ?$ is calculated by performing $3 + ? = 7$</p>  <p>Use cubes to build towers or make bars to find the difference</p>  <p>Use basic bar models with items to find the difference.</p>	 <p>Count on to find the difference.</p> <p>Comparison Bar Models</p> <p>Draw bars to find the difference between 2 numbers.</p> <p><i>Lisa is 13 years old. Her sister is 22 years old. Find the difference in age between them.</i></p>   <p>7 'Seven is 3 more than four'</p> <p>4 'I am 2 years older than my sister'</p>	<p>Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.</p>

Year 1 Subtract from numbers up to 20

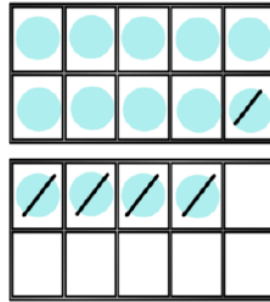
Concrete	Pictorial	Abstract
<p>Revisit / Build on skills taught in Reception.</p> <p>Children consolidate understanding of subtraction practically, showing subtraction on bead strings, using cubes, number lines, on a hundred square etc. and in familiar contexts.</p> <p>Link to addition – use the part-part-whole model to help explain the inverse between addition and subtraction.</p> <p>If the whole is 10 and 6 is one of the parts, what is the other part? $10 - 6 = ?$</p> 	<p>Use pictorial representations of objects to show the part-part-whole model</p> 	<p>Move to using numbers in the part-part-whole model.</p> <p><i>There are eight flowers. Two are red and the rest are yellow. How many are yellow?</i></p> 

Making 10 using ten frames

$14 - 5$



Children to present the ten frame pictorial and discuss what they did to make 10.



Children to show how they can make 10 by partitioning the subtrahend.

$$14 - 5 = 9$$

$$14 - 4 = 10$$

$$10 - 1 = 9$$

Mental subtraction

Children should start recalling subtraction facts up to **and within** 10 and 20, and should be able to subtract zero.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_?

First...then...now...

Key skills for subtraction at Y1:

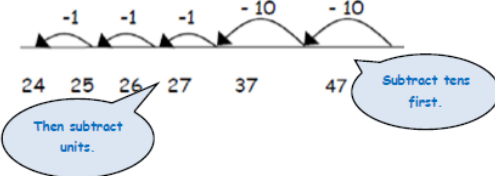
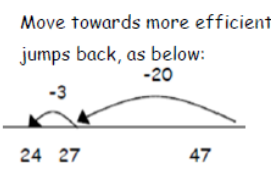
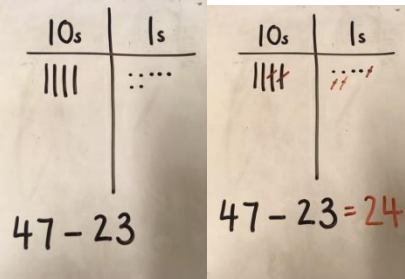
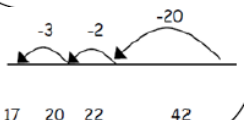
- Given a number, say **one more or one less**.
- Count to and over 100, **forward and back**, from any number.
- Represent and use **subtraction facts to 20 and within 20**.
- Subtract with **one-digit and two-digit** numbers to 20, including zero.
- Solve one-step problems that involve addition and subtraction, using concrete objects (ie bead string, objects, cubes) and pictures, and missing number problems.
- Read and write numbers from 0 to 20 in numerals and words.

Year 2 Subtract with two-digit numbers

Subtract on a number line by counting back, aiming to develop mental subtraction skills.

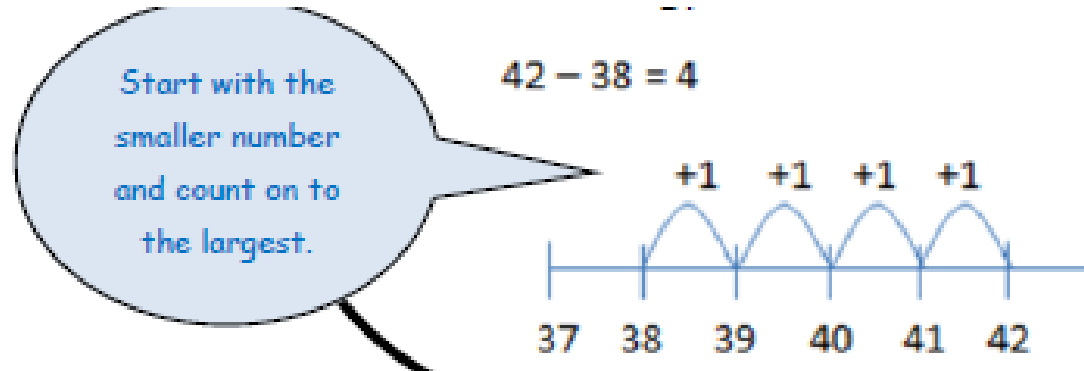
This strategy will be used for:

- 2-digit numbers subtract units (by taking away / counting back) e.g. $36 - 7$
- 2-digit numbers subtract tens (by taking away / counting back) e.g. $48 - 30$
- Subtracting pairs of 2-digit numbers (see below:)

Concrete	Pictorial	Abstract						
<p>Subtracting pairs of 2-digit numbers on a number line</p> <p>Use the dienes alongside the number line to model partitioning.</p> <p>NB When subtracting, you do not need two sets of dienes. It needs to be drawn out that the subtraction is FROM the original number. There are not two numbers.</p> <div style="border: 1px solid black; padding: 5px; margin: 10px 0;"> <p>Combine methods with use of a hundred square to reinforce understanding of number value and order.</p> </div> <p>Ten frames / bead strings can still be used here, especially when doing 2-digit subtract units where they have to bridge through 10. This links to 'making 10' from Y1.</p> <p>$36 - 7$</p> <p>$36 - 6 = 30$</p> <p>$30 - 1 = 29$</p>	<p>$47 - 23 = 24$ Partition the second number and subtract it in tens and units, as below:</p>  <p>Then subtract units. Subtract tens first.</p> <p>Move towards more efficient jumps back, as below:</p>  <p>Children to draw the dienes and use a different colour to cross out.</p>  <p>Teaching children to bridge through ten can help them to become more efficient, for example $42 - 25$:</p> 	<p>Ensure that you present children with a variety of questions: TAKE AWAY (REDUCTION) AND DIFFERENCE.</p> <p>Use the bar model:</p> <table border="1" style="width: 100%; text-align: center;"> <tr> <td colspan="2">48</td> </tr> <tr> <td colspan="2">47</td> </tr> <tr> <td>23</td> <td>?</td> </tr> </table>	48		47		23	?
48								
47								
23	?							

Mental strategy – subtract numbers close together by counting on

Many mental strategies are taught. Children are taught to recognise that when numbers are close together, it is more efficient to **count on** the difference. They need to be clear about the relationship between addition and subtraction.



Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? **difference, count on, strategy, partition, tens, units**

Key skills for subtraction at Y2:

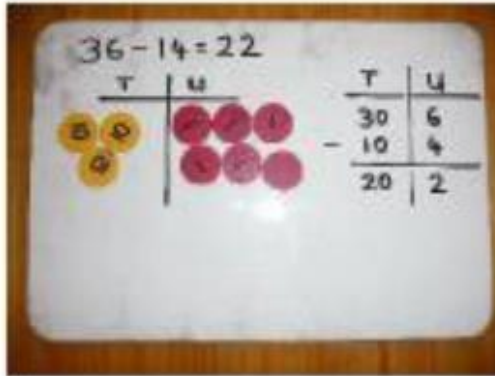
- Recognise the place value of each digit in a two-digit number.
- Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100.
- Subtract using concrete objects, pictorial representations, 100 squares and mentally, including: a two-digit number and ones, a two-digit number and tens, and two two-digit numbers.
- Show that subtraction of one number from another cannot be done in any order.
- Recognise and use inverse relationship between addition and subtraction, using this to check calculations and missing number problems.
- Solve simple addition and subtraction problems including measures, using concrete objects, pictorial representation, and also applying their increasing knowledge of mental and written methods.
- Read and write numbers to at least 100 in numerals and in words.

Year 3 Subtract with two and three-digit numbers

Step 1: Introduce **partitioned column subtraction** method where NO exchanging is required.

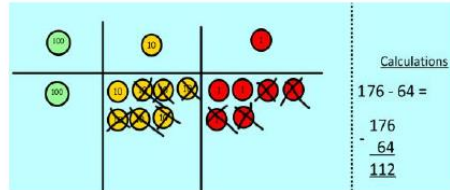
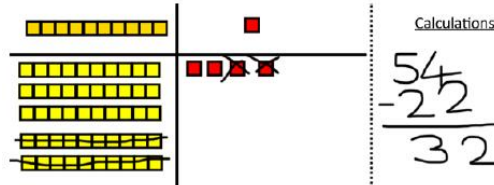
Concrete

Use dienes / PV counters to make the bigger number. Draw out that this is because you are subtracting an amount from this number. It is the WHOLE. You are subtracting one of the parts to find the other part.



Pictorial

Draw the dienes or PV counters alongside the written calculation to help show working.



Abstract

$$89 - 35 = \underline{54}$$

$$\begin{array}{r} 80 + 9 \\ - 30 + 5 \\ \hline 50 + 4 \end{array}$$

$$47 - 24 = 23$$

$$\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$$

Subtraction – Year 3

Step 2: introduce “exchanging” (regrouping) through practical subtraction.

Concrete

Use Base 10 **to start with** before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters. Draw out that this is because you are subtracting an amount from this number. It is the **WHOLE**. You are subtracting one of the parts to find the other part.

$$72 - 47$$



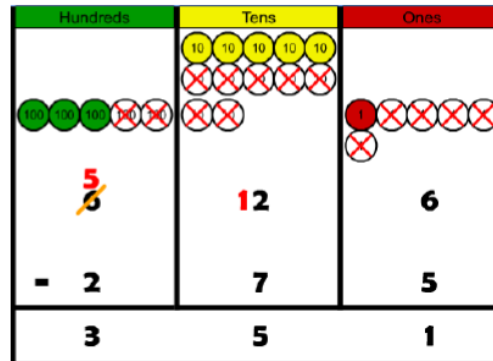
Before subtracting ‘7’ from the 72 blocks, they will need to exchange a row of 10 for ten units. Then subtract 7, and subtract 4 tens.

100	10	1	Calculations
100 100	10 10 10	1 1 1 1	
			234
			- 88

Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.

Pictorial

Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make.



Continue to use the bar model to represent the problems.

626	
275	351

Abstract

Children can start their formal written method by partitioning the number into clear place value columns.

$$\begin{array}{r}
 60 \\
 \cancel{70} + 12 \\
 - 40 + 7 \\
 \hline
 20 + 5 = \underline{25}
 \end{array}$$

When learning to “exchange”, explore “partitioning in different ways” so that pupils understand that when you exchange, the **VALUE** is the same ie $72 = 70+2 = 60+12 = 50+22$ etc. Emphasise that the **value hasn’t changed**, we have just partitioned it in a different way.

Please see:

<https://www.youtube.com/watch?v=dP8NI FLzOg>

Subtraction – Year 3

Step 3: Once pupils are secure with the understanding of “exchanging”, they can use the partitioned column method to subtract any 2 and 3-digit numbers.

Abstract

$$\begin{array}{r}
 238 - 146 = 92 \\
 \hline
 \begin{array}{r}
 \overset{100}{\cancel{200}} + 30 + 8 \\
 - 100 + 40 + 6 \\
 \hline
 0 + 90 + 2
 \end{array}
 \end{array}$$

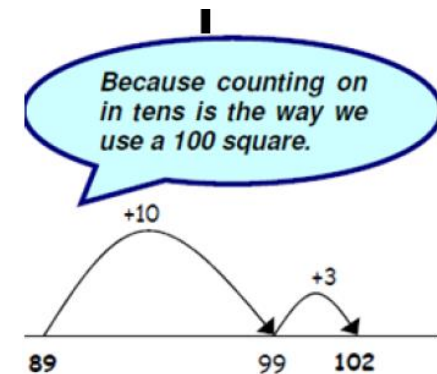
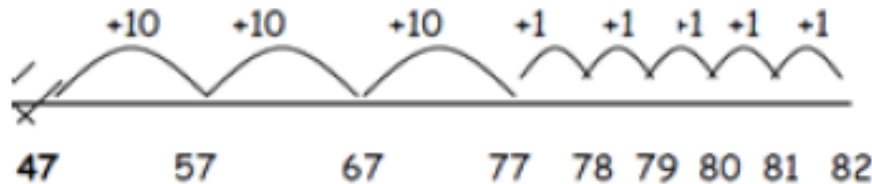
$$\begin{array}{r}
 836 - 254 = 582 \\
 \begin{array}{r}
 \overset{H}{\cancel{800}} \quad \overset{T}{130} \quad \overset{U}{6} \\
 - 200 \quad 50 \quad 4 \\
 \hline
 500 \quad 80 \quad 2
 \end{array}
 \end{array}$$

Subtracting money:
partition into e.g. £1 +
30p + 8p

Counting on as a mental strategy for subtraction:

Continue to reinforce counting **on** as a strategy for **close-together numbers** (e.g. 121—118), and also for numbers that are “nearly” multiples of 10, 100, 1000 or £s, which make it easier to count on (e.g. 102-89, 131—79, or calculating change from £1 etc.).

Start at the smaller number and count on in **tens** first, then count on in units to find the rest of the difference:

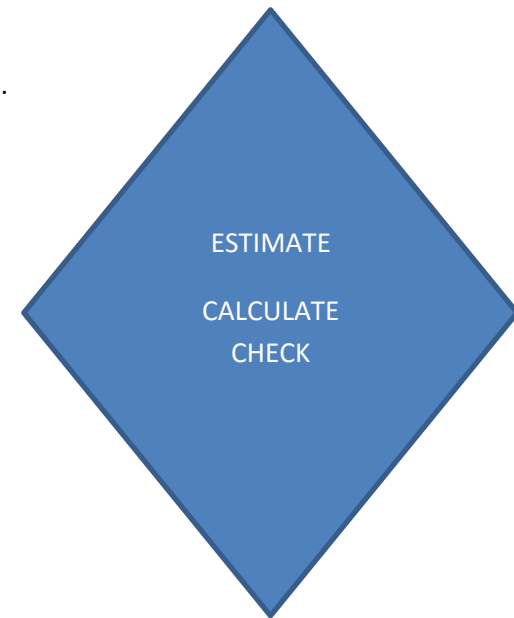


Because counting on in tens is the way we use a 100 square.

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units **exchange, decrease, hundreds, value, digit**

Key skills for subtraction at Y3:

- Subtract mentally a: **3-digit number and ones, 3-digit number and tens, 3-digit number and hundreds** .
- Estimate answers and use inverse operations to check.
- Solve problems, including missing number problems.
- Find 10 or 100 more or less than a given number.
- Recognise the place value of each digit in a 3-digit number .
- Counting up differences as a mental strategy when numbers are close together or near multiples of 10 (see examples above)
- Read and write numbers up to 1000 in numerals and words.
- Practise mental subtraction strategies, such as subtracting near multiples of 10 and adjusting (e.g. subtracting 19 or 21), and select most appropriate methods to subtract, explaining why.



Video clips

<https://www.youtube.com/watch?v=RCCLseBLBS0>

<https://www.youtube.com/watch?v=dP8NIFLZzOg>

Year 4 Subtract with up to four-digit numbers

Concrete	Pictorial	Abstract				
<p>As introduced in Y3, but moving towards more complex numbers and values.</p> <p>Use dienes / PV counters to reinforce 'exchange'.</p>	<p>Continue to use the bar model to represent the problems.</p> <table border="1" data-bbox="815 338 1330 421"> <tr> <td colspan="2" style="text-align: center;">626</td> </tr> <tr> <td style="text-align: center;">275</td> <td style="text-align: center;">351</td> </tr> </table> <p>Continue to use a numberline for finding the difference.</p> <p>Continue to draw dienes / PV counters (see Y3)</p>	626		275	351	<p>Partitioned column subtraction with "exchanging" (decomposition):</p> $\begin{array}{r} 2754 - 1562 = 1192 \\ \hline 2000 + 700 + 50 + 4 \\ - 1000 + 500 + 60 + 2 \\ \hline 1000 + 100 + 90 + 2 \end{array}$ <p style="text-align: center;">↓</p> <p>Follow this with compact column subtraction. Please see: https://www.youtube.com/watch?v=3ihxp2mqnhs</p> <p>To introduce the compact method, ask children to perform a subtraction calculation with the familiar partitioned column subtraction then display the compact version for the calculation they have done. Ask pupils to consider how it relates to the method they know, what is similar and what is different, to develop an understanding of it (shown on video).</p> $\begin{array}{r} 2754 \\ - 1562 \\ \hline 1192 \end{array}$
626						
275	351					

Give plenty of opportunities to apply this to money and measures.

Mental strategies

A variety of mental strategies must be taught and practised, including counting on to find the difference where numbers are closer together, or where it is easier to count on.

Always encourage children to consider the best method for the numbers involved—mental, counting on, counting back or writ-ten method. Please see:

<https://www.youtube.com/watch?v=RCCLseBLBS0>

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, **inverse**

Key skills for subtraction at Y4:

- Subtract by counting on where numbers are close together or they are near to multiples of 10, 100 etc.
- Children select the most appropriate and efficient methods for given subtraction calculations.
- Estimate and use inverse operations to check answers.
- Solve addition and subtraction 2-step problems, choosing which operations and methods to use and why.
- Solve simple measure and money problems involving fractions and decimals to two decimal places.
- Find 1000 more or less than a given number.
- Count backwards through zero, including negative numbers.
- Recognise place value of each digit in a 4-digit number Round any number to the nearest 10, 100 or 1000
- Solve number and practical problems that involve the above, with increasingly large positive numbers.

Year 5 Subtract with at least four-digit numbers including money, measures, decimals.

Subtraction – Year 5

Concrete	Pictorial	Abstract
<p>Use PV counters</p> <div data-bbox="215 456 813 770" style="border: 1px solid black; border-radius: 15px; padding: 10px; background-color: #e1eef6; margin: 10px;"> <p>Children who are still not secure with number facts and place value will need to remain on the partitioned column method (see Y4) until ready for the compact method.</p> </div>	<p>Draw PV counter s</p> <div data-bbox="898 1075 1391 1388" style="border: 1px solid black; border-radius: 15px; padding: 10px; background-color: #e1eef6; margin: 10px;"> <p>Add a “zero” in any empty decimal places to aid understanding of what to subtract in that column.</p> </div>	<p>Compact column subtraction (with “exchanging”) and using larger integers.</p> <p>See video: moving to the compact method https://www.youtube.com/watch?v=3ihxp2mqnhs</p> <div data-bbox="1397 563 1845 775" style="border: 1px solid black; padding: 5px; margin: 10px;"> $\begin{array}{r} \overset{2}{\cancel{3}} \overset{10}{\cancel{1}} \overset{0}{\cancel{0}} \overset{4}{\cancel{5}} \overset{6}{\cancel{6}} \\ - \quad \quad 2 \quad 1 \quad 2 \quad 8 \\ \hline 2 \quad 8 \quad 9 \quad 2 \quad 8 \end{array}$ </div> <p>Subtract with decimal values, including mixtures of integers and decimals, aligning the decimal point.</p> <div data-bbox="1388 954 1845 1193" style="border: 1px solid black; padding: 5px; margin: 10px;"> $\begin{array}{r} \overset{6}{\cancel{7}} \overset{10}{\cancel{1}} \overset{6}{\cancel{6}} \overset{8}{\cancel{9}} \cdot \overset{0}{\cancel{0}} \\ - \quad \quad 3 \quad 7 \quad 2 \cdot 5 \\ \hline 6 \quad 7 \quad 9 \quad 6 \cdot 5 \end{array}$ </div> <p>Create lots of opportunities for subtracting and finding differences with money and measures.</p>

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, **tenths, hundredths, decimal point, decimal**

Key skills for subtraction at Y5:

- Subtract numbers mentally with increasingly large numbers .
- Use rounding and estimation to check answers to calculations and determine, in a range of contexts, levels of accuracy .
- Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.
- Read, write, order and compare numbers to at least 1 million and determine the value of each digit.
- Count forwards or backwards in steps of powers of 10 for any given number up to 1 million.
- Interpret negative numbers in context, counting forwards and backwards with positive and negative integers through 0.
- Round any number up to 1 million to the nearest 10, 100, 1000, 10 000 and 100 000.

Subtraction – Year 6

Year 6 Subtract with increasingly large and more complex numbers and decimal values.

Concrete	Pictorial	Abstract
<p>Use PV counters</p>	<p>Draw PV counters</p>	<p>Using the compact column method to subtract more complex integers</p> $\begin{array}{r} \cancel{9}^{\circ} \cancel{8}^{\circ} \cancel{10}^{\circ} \overset{2}{,} 6 \ 9 \ 9 \\ - \quad 8 \ 9 \ , \ 9 \ 4 \ 9 \\ \hline 6 \ 0 \ , \ 7 \ 5 \ 0 \end{array}$ <p>Using the compact column method to subtract money and measures, including decimals with different numbers of decimal places.</p> $\begin{array}{r} \cancel{10}^{\circ} \overset{2}{,} 5 \ . \ \cancel{4}^{\circ} 1 \ 9 \ \text{kg} \\ 3 \ 6 \ . \ 0 \ 8 \ 0 \ \text{kg} \\ \hline 6 \ 9 \ . \ 3 \ 3 \ 9 \ \text{kg} \end{array}$ <p>Add a “zero” in any empty decimal places to aid understanding of what to subtract in that column.</p> <p>Pupils should be able to apply their knowledge of a range of mental strategies, mental recall skills, and informal and formal written methods when selecting the most appropriate method to work out subtraction problems.</p>

Key vocabulary: equal to, take, take away, less, minus, subtract, leaves, distance between, how many more, how many fewer / less than, most, least, count back, how many left, how much less is_? difference, count on, strategy, partition, tens, units exchange, decrease, hundreds, value, digit, inverse, tenths, hundredths, decimal point, decimal

Key skills for subtraction at Y6:

Solve addition and subtraction multi-step problems in context, deciding which operations and methods to use and why.

Read, write, order and compare numbers up to 10 million and determine the value of each digit

Round any whole number to a required degree of accuracy

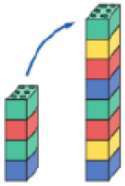

Use negative numbers in context, and calculate intervals across zero.

Children need to utilise and consider a range of mental subtraction strategies, jottings and written methods before choosing how to calculate.




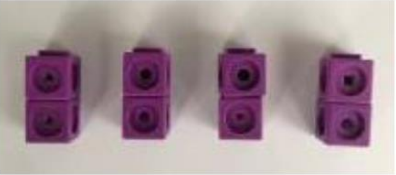

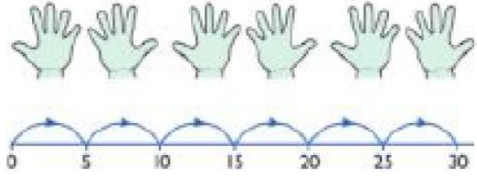
See previous videos for introducing the compact column method.

Reception Double a number.

- Understand that doubling is adding the same number to itself
- In real life contexts, use practical equipment to identify the doubles of numbers up to 5

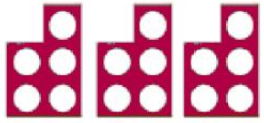
Concrete	Pictorial	Abstract
<p>Use practical activities to show how to double a number.</p> <p>Children to recognise where two groups are the same. Provide patterns of numbers which are doubles, e.g. on dice, dominoes, egg boxes etc.</p> <p>Children describe 'doubles' patterns e.g. 'there are 5 here and 5 here' • discussing what is the same and different about domino and array patterns.</p> <p>Adding</p> <ul style="list-style-type: none"> • adding on the same number again to double • 'counting all' to find the total • 'counting on' from the first number to find the total  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to represent the objects.</p> <p style="text-align: center;">Double 4 is 8</p> 	<p>Double 4 is 8 $4 + 4 = 8$</p>

Year 1 Multiply with concrete objects, arrays and pictorial representations.

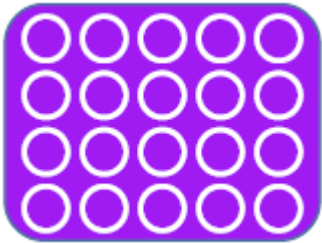
Recall and use doubles of all numbers to 10		
Concrete	Pictorial	Abstract
<p>As Reception – continue to use concrete objects to recall and use doubles of all numbers up to 10.</p>	<p>Draw pictures to represent the objects. Double 4 is 8</p>  <p>Children to draw arrays to show doubles.</p>	<p>Double 4 is 8 $4 + 4 = 8$</p> <p>Introduce the 'x' symbol. $4 \times 2 = 8$ $2 \times 4 = 8$</p> <p>How many different number sentences can you make using this array?</p> 
Counting in multiples		
<p>Give children experience of counting equal group of objects in 2s, 5s and 10s.</p> <p>Count in multiples supported by concrete objects in equal groups.</p>   <p>How many legs will 3 teddies have?</p>  $2 + 2 + 2 = 6$	<p>Use a number line or pictures to continue support in counting in multiples.</p> 	<p>Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30</p>

Repeated addition

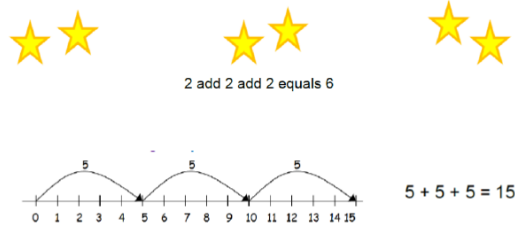
Use different objects to add equal groups.



There are 3 plates. Each plate has 2 star biscuits on it. How many biscuits are there?



This is a baking tray for making buns.
 How many buns can fit on the top row?
 How many on the second row? Third row? Fourth row?
 The answer to all of these is 5 giving rise to the sum:
 $5 + 5 + 5 + 5$



Write addition sentences to describe objects and pictures.



$$2 + 2 + 2 + 2 + 2 = 10$$

Present practical problem solving activities involving counting equal sets or groups, as above.

Arrays- showing commutative multiplication

Create arrays using counters/ cubes to show multiplication sentences.



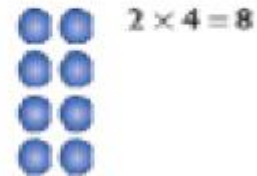
Use an array-finder (two strips of card stapled together at a right angle).

Give children a piece of paper with large squares on it. Place the array-finder onto the grid to create an array. EG. Find the array that shows 2 x 6. Is yours the same as your partner's? Are you both correct? Why? → COMMUTATIVE MULTIPLICATION

Draw arrays in different rotations to find **commutative** multiplication sentences.



$$2 \times 4 = 8$$



$$4 \times 2 = 8$$

Use an array to write multiplication sentences and reinforce repeated addition.



$$5 + 5 + 5 = 15$$

$$3 + 3 + 3 + 3 + 3 = 15$$

$$5 \times 3 = 15$$

$$3 \times 5 = 15$$

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count

multiplication sign is equivalent to

$$4 \times 5 = 20$$

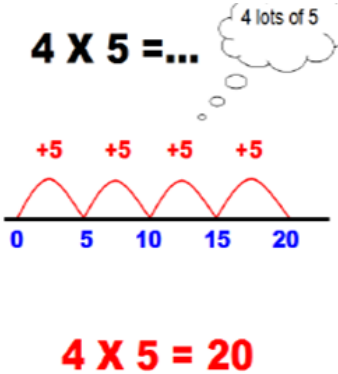


factor factor product



Multiplication (commutative)

Key skills for multiplication at Y1:

- Count in multiples of 2, 5 and 10.
- Solve one-step problems involving multiplication, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
- Make connections between arrays, number patterns, and counting in twos, fives and tens.
- Begin to understand doubling using concrete objects and pictorial representations.

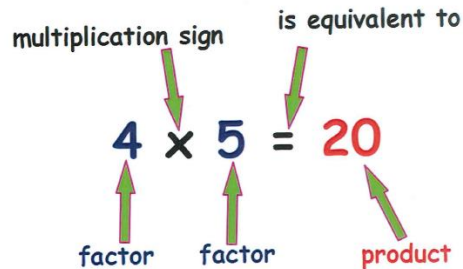
Year 2 Multiply using arrays and repeated addition (using at least 2s, 5s and 10s).

Use repeated addition on a number line		
Concrete	Pictorial	Abstract
<p>Children should use their of counting in equal steps in the context of money as a way of understanding multiplication as repeated addition. They should explore repeated addition practically making groups of equal size and also through the model of an array.</p>	<p>Starting from zero, make equal jumps up on a number line to work out multiplication facts and write multiplication statements using x and = signs.</p>  <p>4 X 5 = 20</p> <p>Tally charts help children see that multiplication is repeated addition. When finding the total, children may initially count but should be encouraged to recognise the multiplication fact $5 \times 5 = 25$</p> 	<p>$4 \times 5 = 20$</p> <p>$5 \times 4 = 20$</p> <p>$5 + 5 + 5 + 5 = 20$</p> <p>$4 + 4 + 4 + 4 + 4 = 20$</p> <p>Children should relate multiplication (as repeated addition) to other familiar contexts such as identifying the time on a clock without having to count round in fives.</p>  <p>When the minute hand is pointing at five, it has moved through five groups of 5 minutes since the last o'clock time. Knowing multiplication facts for the 5x table supports fluent telling of time and solving some time problems.</p>

Use arrays						
Concrete	Pictorial	Abstract				
<p>Create arrays using counters/ cubes to show multiplication sentences.</p>   <p>Use an array-finder (two strips of card stapled together at a right angle). Give children a piece of paper with large squares on it. Place the array-finder onto the grid to create an array. EG. Find the array that shows 2 x 6. Is yours the same as your partner's? Are you both correct? Why? → COMMUTATIVE MULTIPLICATION</p> <p>Please see the following video: practical multiplication and the commutative law. https://www.youtube.com/watch?v=VGkjjVfnGYI&list=PLQqF8sn28L9yj34NpXK7Yffze7ZoXTiix&index=2</p>	<p>Children to draw the arrays.</p> <p>THEN use bar modelling. ONE BAR then represents 5.</p> <table border="1" data-bbox="815 434 1348 587"> <tbody> <tr><td>5</td></tr> <tr><td>5</td></tr> <tr><td>5</td></tr> <tr><td>5</td></tr> </tbody> </table> <p>$5 \times 4 = 20$</p>	5	5	5	5	<p>Rote counting should be linked to repeated addition and the creation of arrays. Children should learn that multiplication is a convenient way of repeatedly adding a number to itself e.g. $2+2+2+2+2$ can be said as 2×6 (2 added to itself 6 times). The array created can then be used to demonstrate commutativity i.e. that 2×6 is the same as 6×2. Children should make links to real life application of multiplication as repeated addition.</p> <p>Children should begin to relate counting in steps of 2, 3, 5 and 10 to the multiplication tables.</p> <p>$4 \times 5 = 20$</p> <p>$5 \times 4 = 20$</p> <p>$5 + 5 + 5 + 5 = 20$</p> <p>$4 + 4 + 4 + 4 + 4 = 20$</p> <p>Ensure that children are given missing number examples such as $3 \times \underline{\quad} = 6$.</p>
5						
5						
5						
5						

Use mental recall		
		<p>Children should begin to recall multiplication facts for 2, 5 and 10 times tables through practice in counting and understanding of the operation.</p> <p>Teaching for understanding of multiplication facts: https://www.youtube.com/watch?v=YPWmOVt8vgw&list=PLQqF8sn28L9yj34NpXK7Yffze7ZoXTiix</p>

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times...

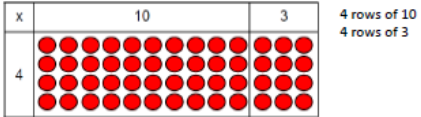
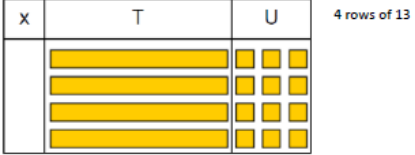
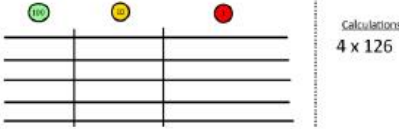
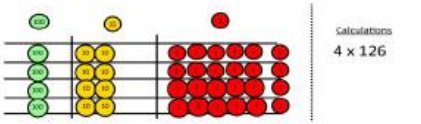
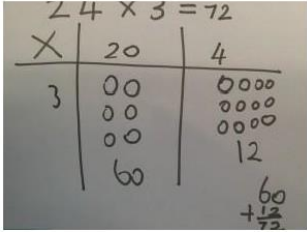
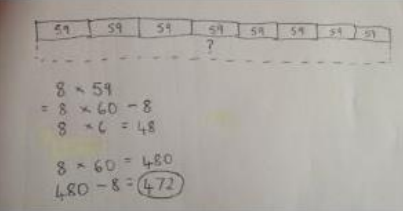
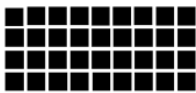
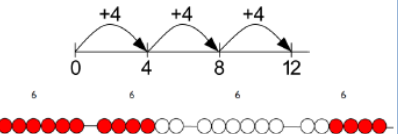


Multiplication (commutative)

Key skills for multiplication at Y2:

- Count in steps of 2, 3 and 5 from zero, and in 10s from any number.
- Recall and use multiplication facts from the 2, 5 and 10 multiplication tables, including recognising odd and evens.
- Write and calculate number statements using the \times and $=$ signs.
- Show that multiplication can be done in any order (commutative).
- Solve a range of problems involving multiplication, using concrete objects, arrays, repeated addition, mental methods, and multiplication facts.
- Pupils use a variety of language to discuss and describe multiplication.

Year 3 Multiply 2-digits by a single digit number.

Concrete	Pictorial	Abstract																														
<p>Show the link with arrays to first introduce the grid method.</p> <p>Introduce the grid method with children physically making an array to represent the calculation (e.g. make 8 lots of 23 with 10s and 1s place value counters), then translate this to grid method format. Please see: https://www.youtube.com/watch?v=qyTRtoqYi7Q&list=PLQqF8sn28L9yj34NpXK7Yffze7ZoXTiix</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move on to using Base 10 to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.</p>  <p>Calculations 4 x 126</p> <p>Fill each row with 126.</p>  <p>Calculations 4 x 126</p> <p>Add up each column, starting with the ones making any exchanges needed.</p>	<p>Children can represent the work they have done with place value counters in a way that they understand. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.</p>  <p>Bar modelling helps to support learners when solving problems with multiplication alongside the formal written methods.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1397 384 1704 475"> <tr> <td>X</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>210 + 35 = 245</p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table border="1" data-bbox="1442 663 1688 823"> <tr> <td></td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td>100</td> <td>80</td> </tr> <tr> <td>3</td> <td>30</td> <td>24</td> </tr> </table> <table border="1" data-bbox="1391 850 1749 986"> <tr> <td>X</td> <td>1000</td> <td>300</td> <td>40</td> <td>2</td> </tr> <tr> <td>10</td> <td>10000</td> <td>3000</td> <td>400</td> <td>20</td> </tr> <tr> <td>8</td> <td>8000</td> <td>2400</td> <td>320</td> <td>16</td> </tr> </table>	X	30	5	7	210	35		10	8	10	100	80	3	30	24	X	1000	300	40	2	10	10000	3000	400	20	8	8000	2400	320	16
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X	1000	300	40	2																												
10	10000	3000	400	20																												
8	8000	2400	320	16																												
<p>To do this, children must be able to:</p> <ul style="list-style-type: none"> • Partition numbers into tens and units • Multiply multiples of ten by a single digit (e.g. 20 x 4) using their knowledge of multiplication facts and place value • Recall and work out multiplication facts in the 2, 3, 4, 5, 8 and 10 times tables. • Work out multiplication facts not known by repeated addition or other taught mental strategies (e.g. by commutative law, working out near multiples and adjusting, using doubling etc.) Strategies to support this are repeated addition using a number line, bead bars and arrays:  <p>9 x 4 = 36</p> 																																

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, column, row, commutative, sets of, equal groups, times, _times as big as, once, twice, three times..., partition, grid method, multiple, product, tens, units, value

multiplication sign is equivalent to

$$4 \times 5 = 20$$

factor factor product

Multiplication (commutative)

Key skills for multiplication:

- Recall and use multiplication facts for the 2, 3, 4, 5, 8 and 10 multiplication tables, and multiply multiples of 10.
- Write and calculate number statements using the multiplication tables they know, including 2-digit x single-digit, drawing upon mental methods, and progressing to reliable written methods.
- Solve multiplication problems, including missing number problems.
- Develop mental strategies using commutativity (e.g. $4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$)
- Solve simple problems in contexts, deciding which operations and methods to use.
- Develop efficient mental methods to solve a range of problems eg. using commutativity ($4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240$) and for missing number problems $? \times 5 = 20$, $3 \times ? = 18$, $? \times ? = 32$

Year 4 Multiply 2 and 3-digits by a single digit, using all multiplication tables up to 12 x 12.

Concrete	Pictorial	Abstract																									
As Y3 - continue to develop the grid method. Use the PV counters / dienes (base 10) equipment.	As Y3 – draw the PV counters or dienes. Use bar modelling to represent the problem.	<p>Eg. $136 \times 5 = 680$</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td></td> <td></td> <td></td> <td></td> <td style="text-align: right;">500</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td style="text-align: right;">150</td> </tr> <tr> <td style="border: 1px solid black;">X</td> <td style="border: 1px solid black;">100</td> <td style="border: 1px solid black;">30</td> <td style="border: 1px solid black;">6</td> <td></td> </tr> <tr> <td style="border: 1px solid black;">5</td> <td style="border: 1px solid black;">500</td> <td style="border: 1px solid black;">150</td> <td style="border: 1px solid black;">30</td> <td style="text-align: right;">+ 30</td> </tr> <tr> <td></td> <td></td> <td></td> <td></td> <td style="text-align: right; border-top: 1px solid black;">680</td> </tr> </table> <p>Encourage column addition to add accurately.</p> <p>Move onto short multiplication (see Y5) if and when children are confident and accurate multiplying 2 and 3-digit numbers by a single digit this way, and are already confident in “carrying” for written addition.</p>					500					150	X	100	30	6		5	500	150	30	+ 30					680
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X	100	30	6																								
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Children should be able to:

- **Approximate before they calculate**, and make this a regular part of their calculating, going back to the approximation to check the reasonableness of their answer. eg:
- “**346 x 9** is approximately $350 \times 10 = 3500$ ”
- Record an approximation to check the final answer against.
- Multiply multiples of ten and one hundred by a single-digit, using their multiplication table knowledge.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, groups of, sets of, lots of, equal groups, times, multiply, times as big as, once, twice, three times... partition, grid method, total, multiple, product, sets of, inverse

multiplication sign is equivalent to

$$4 \times 5 = 20$$

factor factor product

Multiplication (commutative)

Key skills for multiplication at Y4:

- Count in multiples of 6, 7, 9, 25 and 1000
- Recall multiplication facts for all multiplication tables up to 12 x 12.
- Recognise place value of digits in up to 4-digit numbers
- Use place value, known facts and derived facts to multiply mentally, e.g. multiply by 1, 10, 100, by 0, or to multiply 3 numbers.
- Use commutativity and other strategies mentally $3 \times 6 = 6 \times 3$, $2 \times 6 \times 5 = 10 \times 6$, $39 \times 7 = 30 \times 7 + 9 \times 7$.
- Solve problems with increasingly complex multiplication in a range of contexts.
- Count in multiples of 6, 7, 9, 25 and 1000
- Recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)

Year 5 Multiply up to 4-digits by 1 or 2 digits.

Introducing column multiplication

Introduce by comparing a grid method calculation to a short multiplication method, to see how the steps are related, but notice how there are less steps involved in the column method (see video).

Children need to be taught to approximate first, e.g. for 72×38 , they will use rounding: 72×38 is approximately $70 \times 40 = 2800$, and use the approximation to check the reasonableness of their answer against.

Short multiplication for multiplying by a single digit

Concrete	Pictorial	Abstract																												
	Use bar modelling.	<p>Pupils could be asked to work out a given calculation using the grid, and then compare it to “your” column method. What are the similarities and differences? Unpick the steps and show how it reduces the steps.</p> <div style="text-align: center;"> <table border="1" style="display: inline-table; margin-right: 20px;"> <tr><td>x</td><td>300</td><td>20</td><td>7</td></tr> <tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr> </table> ➔ <table border="1" style="display: inline-table; margin-left: 20px;"> <tr><td></td><td>3</td><td>2</td><td>7</td></tr> <tr><td>x</td><td></td><td></td><td>4</td></tr> <tr><td></td><td>1</td><td>3</td><td>0</td></tr> <tr><td></td><td></td><td>1</td><td>2</td></tr> <tr><td></td><td></td><td></td><td>8</td></tr> </table> </div>	x	300	20	7	4	1200	80	28		3	2	7	x			4		1	3	0			1	2				8
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Multiplication – Year 5

Introduce long multiplication for multiplying by 2 digits, using the grid first		
Concrete	Pictorial	Abstract
		<p>The grid could be used to introduce long multiplication, as the relationship can be seen in the answers in each row.</p>

18 x 3 on the 1st row
 (8 x 3 = 24, carrying the 2 for twenty, then "1" x 3).
 18 x 10 on the 2nd row. Put a zero in units first, then say 8 x 1, and 1 x 1.

Moving towards more complex numbers:		
Concrete	Pictorial	Abstract

ESTIMATE
 CALCULATE
 CHECK

Key vocabulary groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated ad-dition, column, row, commutative, sets of, equal groups, _times as big as, once, twice, three times..., parti-tion, grid method, total, multiple, product, inverse, **square, factor, integer, decimal, short/long multiplication, 'carry'**

multiplication sign is equivalent to

$$4 \times 5 = 20$$

factor factor product

Multiplication (commutative)

Key skills for multiplication at Y5:

- Identify multiples and factors, using knowledge of **multiplication tables to 12x12**.
- Solve problems where larger numbers are decomposed into their factors
- Multiply and divide integers and decimals by 10, 100 and 1000
- Recognise and use square and cube numbers and their notation
- Solve problems involving combinations of operations, choosing and using calculations and methods appropriately.

Video clips:

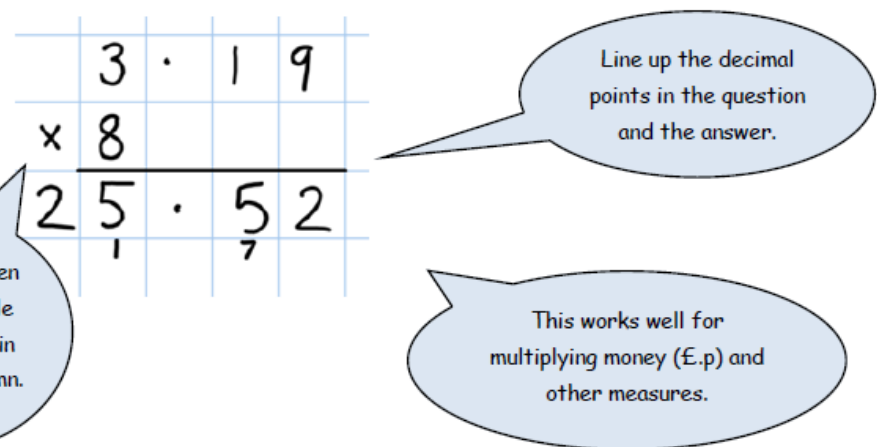
[Moving from grid method to a compact method](#)

[Reinforcing rapid times table recall](#)

[Demonstration of long multiplication](#)

Multiplication – Year 6

Year 6 Short and long multiplication as in Y5, and multiply decimals with up to 2d.p by a single digit.

Concrete	Pictorial	Abstract
	Use bar modelling.	 <p>Remind children that the single digit belongs in the units column.</p> <p>Line up the decimal points in the question and the answer.</p> <p>This works well for multiplying money (£.p) and other measures.</p> <p>This can also be solved: 319×8 Explain to children that, to get a whole number, they need to multiply by a power of 10 – in this case, 100. Their answer will, therefore, be 100 times too big. They will need to divide their answer by 100.</p>

Children will be able to:

- Use rounding and place value to make approximations before calculating and use these to check answers against.
- Use **short multiplication** (see Y5) to multiply numbers with **more than 4-digits by a single digit**; to multiply money and measures, and to **multiply decimals with up to 2d.p. by a single digit**.
- Use **long multiplication** (see Y5) to multiply numbers with **at least 4 digits by a 2-digit number**.

Key vocabulary: groups of, lots of, times, array, altogether, multiply, count, multiplied by, repeated addition, array, column, row, commutative, sets of, equal groups, times as big as, once, twice, three times... partition, grid method, total, multiple, product, inverse, square, factor, integer, decimal, short / long multiplication, "carry", **tenths, hundredths, decimal**

multiplication sign is equivalent to

$$4 \times 5 = 20$$

factor factor product

Multiplication (commutative)

Key skills for multiplication at Y6:

Recall multiplication facts for all times tables up to **12 x 12 (as Y4 and Y5)**.

Multiply multi-digit numbers, up to 4-digit x 2-digit using long multiplication.

Perform mental calculations with mixed operations and large numbers.

Solve multi-step problems in a range of contexts, choosing appropriate combinations of operations and methods.

Estimate answers using round and approximation and determine levels of accuracy.

Round any integer to a required degree of accuracy.

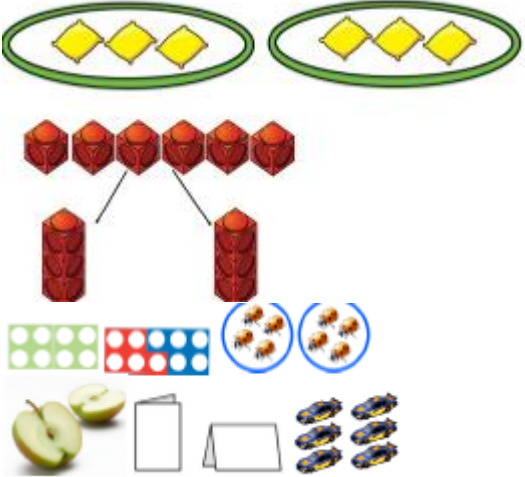
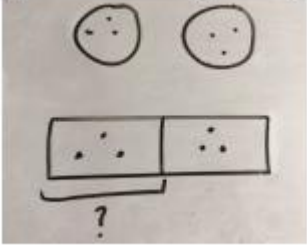
Video clips:

[Moving from grid method to a compact method \(youtube\)](#)

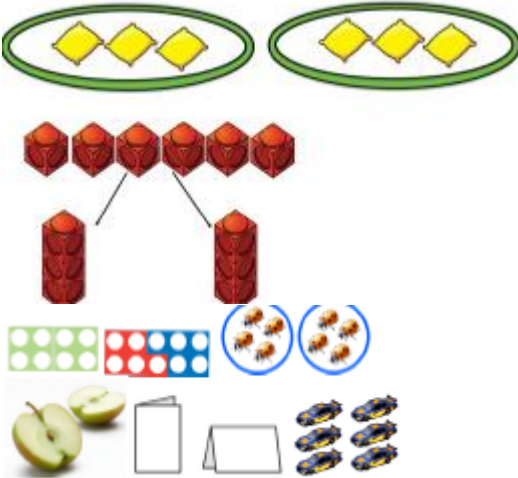

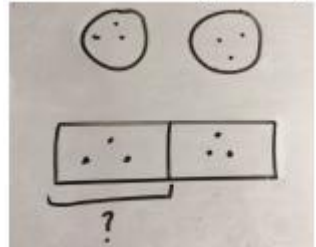
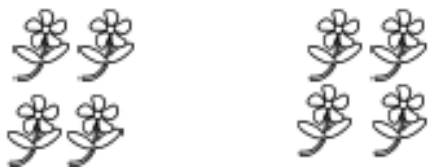
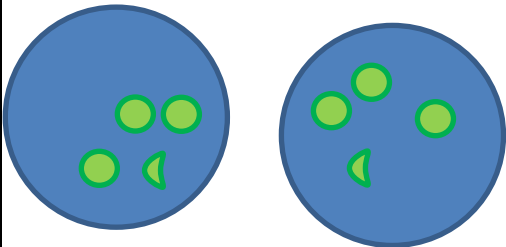
[Reinforcing rapid times table recall: \(youtube\)](#)

[Demonstration of long multiplication \(SLEP\)](#)

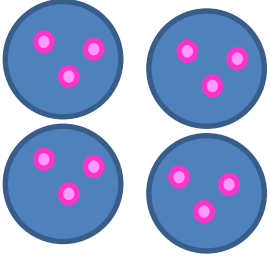
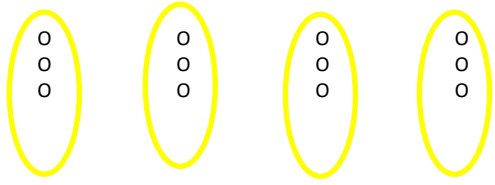
Reception Halving a number.

Concrete	Pictorial	Abstract
<p>Use a range of objects to explore:</p> <ul style="list-style-type: none"> • Understand that when an amount has been shared equally all parts are the same • Recognise, by counting, whether an amount has been shared equally or not • In real life contexts, use practical equipment and equal sharing to find one half of an even amount of objects • Understand that the terms halving and sharing between two relate to splitting into two equal sized parts 	<p>Represent the sharing pictorially.</p> 	<p>Halving means sharing (dividing) into two equal groups.</p> <p>If I share 6 into two equal groups, there are 3 in each group.</p> <p>Half of 6 is 3.</p> <p>6 divided by two is 3.</p>

Year 1 Recall and use halves of all numbers to 10.

Concrete	Pictorial	Abstract
<p>Use a range of objects to explore:</p> <ul style="list-style-type: none"> halves as splitting a group into two equal parts Recall halves for even numbers to ten using finger patterns to support if required Recall halves for even numbers from 12 to 20 Find a half of an even quantity Find half of an odd quantity using materials that can be cut e.g. grapes or buns   <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Represent the sharing pictorially.</p>  <p>Children use pictures or shapes to share quantities.</p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 0 auto;"> $8 \div 2 = 4$ </div> <p>Share 7 apples between two people</p> 	<p>Halving means sharing (dividing) into two equal groups.</p> <p>If I share 6 into 2 equal groups, there are 3 in each group.</p> <p>Half of 6 is 3.</p> <hr/> $6 \div 2 = 3$ <div style="border: 1px solid black; display: inline-block; padding: 5px; margin: 5px;">3</div> <div style="border: 1px solid black; display: inline-block; padding: 5px; margin: 5px;">3</div> <p>Children should also be encouraged to use their 2 times tables facts.</p> <p>7 apples shared between two people is $3 \frac{1}{2}$ apples each.</p>

Year 1 Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity (*including measure*)

Concrete	Pictorial	Abstract
<p>Use a range of objects to explore:</p> <ul style="list-style-type: none"> Find a quarter of an object (using objects that can be accurately quartered e.g. a KitKat) Recognise and name a quarter as one of four equal parts of a quantity (which is a multiple of 4) Find a quarter of a quantity Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity (<i>including measure</i>) <p>SHARING There are 12 cakes and I share them out onto 4 plates. How many cakes will there be on each plate? Use cakes and plates to model. Then use cubes / counters to represent the object.</p>	<p>Represent the problem pictorially.</p> <p>SHARING There are 12 cakes and I share them out onto 4 plates. How many cakes will there be on each plate?</p>  <p>GROUPING There are 12 cakes in my tin. I want to put 3 in each box. How many boxes will I need? How many groups of 3 will I have?</p> 	$12 \div 4 = 3$

Key Vocabulary: share, share equally, one each, two each..., group, groups of, lots of, array




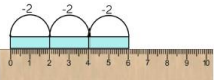
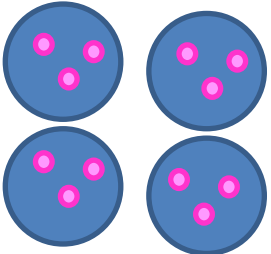
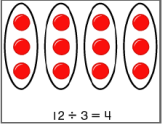
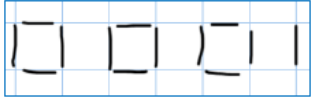
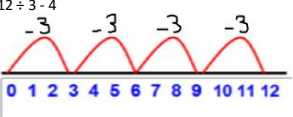
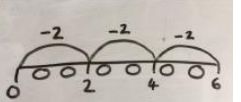
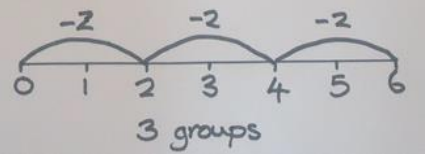
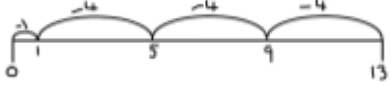

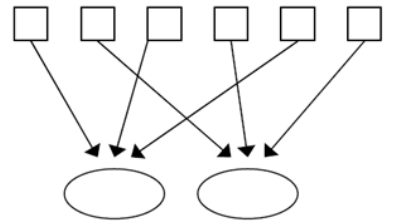
The diagram shows the equation $12 \div 4 = 3$. The number 12 is in red, 4 is in blue, and 3 is in green. The division sign \div is black, and the equals sign $=$ is black. Labels with arrows point to each part: 'dividend' (red) points to 12, 'divisor' (blue) points to 4, and 'quotient' (green) points to 3. Above the division sign is the label 'division sign' with a green arrow pointing to it. Above the equals sign is the label 'is equivalent to' with a green arrow pointing to it.

Division

Key number skills needed for division at Y1:

- Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations arrays with the support of the teacher
- Through grouping and sharing small quantities, pupils begin to understand, division, and finding simple fractions of objects, numbers and quantities.
- They make connections between arrays, number patterns, and counting in twos, fives and tens.

Year 2 Group and share, using the ÷ and = sign

Concrete	Pictorial	Abstract
<p>Use a range of objects to explore:</p> <ul style="list-style-type: none"> Share an amount equally across sets where there is no remainder e.g. share 20 sweets between 5 children In real life contexts, share an amount equally across sets where there is a remainder e.g. share 23 pencils between 3 tables results in 7 pencils on each table and 2 pencils that cannot be shared Make equal sized groups from an amount where there is no remainder e.g. make teams of 5 from a group of 30 children; $24 \div 6$ Make equal sized groups from an amount where there is a remainder e.g. give 3 buttons to each gingerbread man when there are 23 buttons in total; $26 \div 5$ <p>SHARING Use concrete objects to explore sharing firstly where there is no remainder and then where there is a remainder.</p> <p>There are 12 cakes and I share them out onto 4 plates. How many cakes will there be on each plate? Use cakes and plates to model. Then use cubes / counters to represent the object.</p> <p>There are 13 flowers. I want to plant them in rows of 4. How many will there be in each row?</p> <p>GROUPING There are 12 cakes in my tin. I want to put a $\frac{1}{4}$ of the total in each box. How many boxes will I need? How many groups will I have? How many will be in each group?</p>  <p>Act out the problem If there are 3 people of each team, how many teams can you have if there are 12 children?</p>  <p>Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.</p>  <p>There are 3 whole squares, with 1 left over.</p> <p>Repeated subtraction</p>  <p>3 groups of 2</p>	<p>Represent the problem pictorially.</p> <p>SHARING There are 12 cakes and I share them out onto 4 plates. How many cakes will there be on each plate?</p>  <p>What about if I have 23 cakes?</p> <p>GROUPING Use arrays This represents $12 \div 3$ posed as how many groups of 3 are there in 12? Pupils should show that the same array can represent $12 \div 4$ if grouped horizontally.</p>  <p>There are 12 cakes in my tin. I want to put 3 in each box. How many boxes will I need? How many groups of 3 will I have?</p> <p>What about if I have 17 cakes and put 3 in each box. How many boxes will I need?</p>  <p>There are three whole squares with one left over.</p> <p>Use a number line</p> <p>$12 \div 3 = 4$</p>   <p>Group from zero in equal jumps of the divisor to find out "how many groups of _ in _?". Pupils could use a bead string or practical apparatus to work out problems like "A CD costs £3. How many CDs can I buy with £12?" This is an important method to develop understanding of division as grouping.</p>	<p>$12 \div 4 = 3$</p> <p>$12 \div 3 = 4$</p> <p>Number line to represent the equal groups that have been subtracted.</p>  <p>$13 \div 4 = 3$ remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line. '3 groups of 4, with 1 left over'</p>  <div style="border: 2px solid blue; border-radius: 25px; padding: 10px; margin-top: 20px;"> <p>Children should be taught to recognise whether problems require sharing or grouping.</p> <p style="text-align: center;">Grouping:</p> <div style="border: 1px solid black; padding: 5px; text-align: center;"> <p>There are 6 sweets, how many people can have 2 sweets each?</p>  </div> <p style="text-align: center;">Sharing:</p>  </div>

Key Vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over

The diagram shows the equation $12 \div 4 = 3$. The number 12 is in red, 4 is in blue, and 3 is in green. The division sign \div and equals sign $=$ are black. Labels with arrows point to each part: 'dividend' (red) points to 12, 'divisor' (blue) points to 4, 'quotient' (green) points to 3, 'division sign' (black) points to \div , and 'is equivalent to' (black) points to $=$.

Division

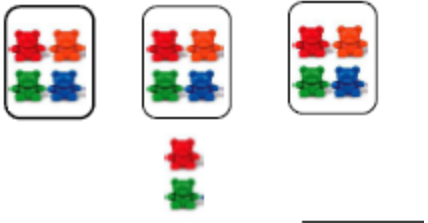
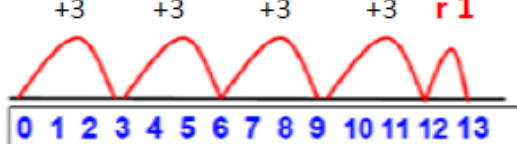


Key number skills needed for division at Y2:

- Count in steps of 2, 3, and 5 from 0
- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.
- Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the \times , \div and $=$ signs.
- Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.
- Solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Year 3 Divide 2-digit numbers by a single digit

Step 1: Grouping on a number line

Children continue to work out unknown division facts by grouping on a number line from zero. They are also now taught the concept of **remainders**, as in the example. This should be introduced practically and with arrays, as well as being translated to a number line. Children should work towards calculating some basic division facts with remainders mentally for the 2s, 3s, 4s, 5s, 8s and 10s, ready for „carrying“ remainders across within the short division method.

Concrete	Pictorial	Abstract
<p>Divide objects between groups and see how much is left over.</p> <p>$14 \div 3$</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p> <p>$13 \div 3 = 4 \text{ r } 1$</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p> 	<p>Complete written divisions and show the remainder using r.</p> <p>$29 \div 8 = 3 \text{ REMAINDER } 5$</p> 

Real life contexts need to be used routinely to help pupils gain a full understanding, and the ability to recognise the place of division and how to apply it to problems.

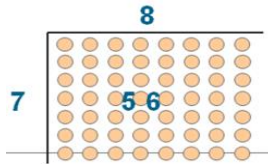
Step 2: Short division

Limit numbers to **NO** remainders in the answer **OR** carried (each digit must be a multiple of the divisor).

Once children are secure with division as grouping and demonstrate this using number lines, arrays etc., **short division** for larger 2-digit numbers should be introduced, initially with carefully selected examples requiring no calculating of remainders at all.

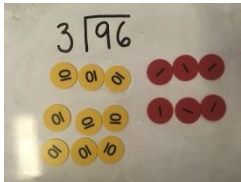
Concrete

Start by introducing the layout of short division by comparing it to an array.

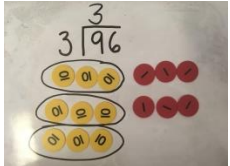


Use PV counters / dienes to introduce the bus stop method.

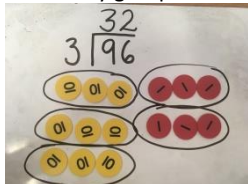
$$96 \div 3$$



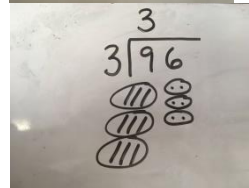
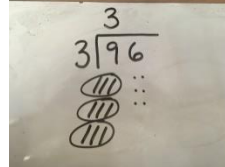
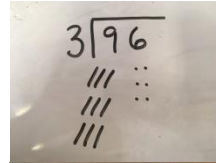
How many groups of 3 can we make with 9 tens?



How many groups of 3 can we make with 6 ones?

**Pictorial**

Children to draw the dienes / PV counters underneath their bus stop calculation.

**Abstract**

Remind children of correct place value, that 96 is equal to 90 and 6, but in short division, pose:

How many groups of 3 are there in 9? = 3, and record it above the 9 tens.

How many groups of 3 are there in 6? = 2, and record it above the 6 units.

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 32 \\ 3 \overline{)96} \end{array}$$

Step 3: Short division

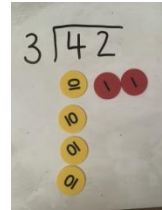
Limit numbers to **NO** remainders in the final answer, but with remainders occurring within the calculation

Once children demonstrate a full understanding of remainders, and also the short division method taught, they can be taught how to use the method when remainders occur within the calculation (e.g. $96 \div 4$), and be taught to „carry“ the remainder onto the next digit. **If needed, children should use the number line to work out individual division facts that occur which they are not yet able to recall mentally.**

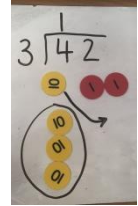
Concrete

Use PV counters / dienes

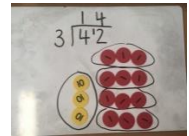
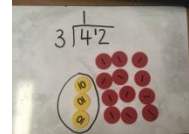
$$42 \div 3 =$$



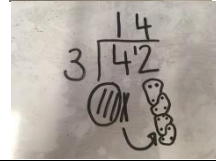
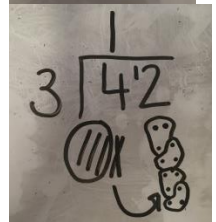
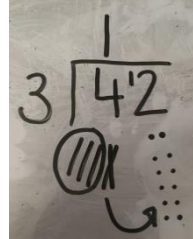
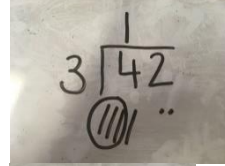
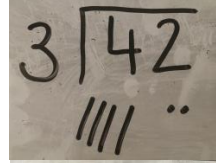
How many groups of 3 can we make with the tens? One group with one ten left over.



We exchange this ten for ten ones and then find out how many groups of 3 we can make with the ones.

**Pictorial**

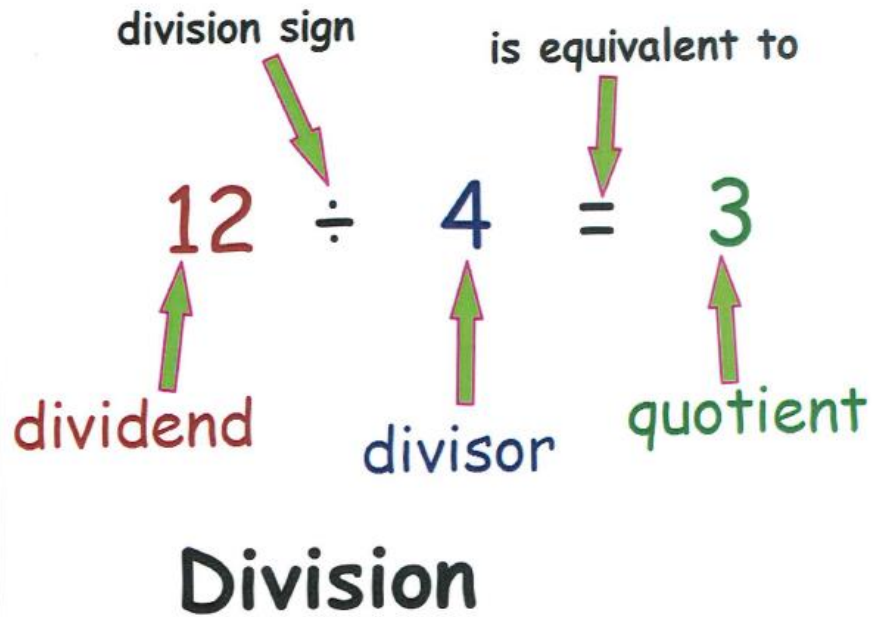
Children to draw the dienes / PV counters underneath their bus stop calculation.

**Abstract**

$$\begin{array}{r} 218 \\ 3 \overline{) 654} \\ \underline{6} \\ 0 \\ \underline{0} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

Key Vocabulary:

share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, 'carry', remainder, multiple

**Key number skills needed for division at Y3:**

- Recall and use multiplication and division facts for the 2, 3, 4, 5, 8 and 10 multiplication tables (through doubling, connect the 2, 4 and 8s).
- Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.
- Solve problems, in contexts, and including missing number problems, involving multiplication and division.
- Pupils develop efficient mental methods, for example, using multiplication and division facts (e.g. using $3 \times 2 = 6$, $6 \div 3 = 2$ and $2 = 6 \div 3$) to derive related facts ($30 \times 2 = 60$, so $60 \div 3 = 20$ and $20 = 60 \div 3$).
- Pupils develop reliable written methods for division, starting with calculations of 2-digit numbers by 1-digit numbers and progressing to the formal written method of short division.

Year 4 Divide up to 3-digit numbers by a single digit (without remainders initially)

Concrete	Pictorial	Abstract
See Y3	See Y3	<p>Short division should only be taught once children have secured the skill of calculating “remainders”.</p> <p>STEP 1: Continue to develop short division. Pupils must be secure with the process of short division for dividing 2-digit numbers by a single digit (those that do not result in a final remainder—see steps in Y3), but must understand how to calculate remainders, using this to “carry” remainders within the calculation process (see example).</p> $\begin{array}{r} 18 \\ 4 \overline{)72} \end{array}$ <p>STEP 2: Pupils move onto dividing numbers with up to 3-digits by a single digit, however problems and calculations provided should not result in a final answer with remainder at this stage. Children who exceed this expectation may progress to Y5 level.</p> $\begin{array}{r} 218 \\ 4 \overline{)872} \end{array}$ <p>NB: When the answer for the first column is zero ($1 \div 5$, as in example), children could initially write a zero above to acknowledge its place, and must always “carry” the number (1) over to the next digit as a remainder.</p> $\begin{array}{r} 037 \\ 5 \overline{)185} \end{array}$

Key Vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, **divisible by, factor**

division sign is equivalent to

$$12 \div 4 = 3$$

dividend divisor quotient

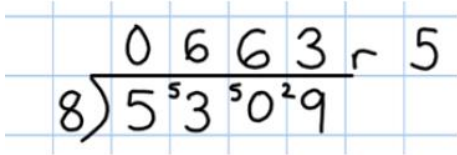
Division

Key number skills needed for division at Y4:

- Recall multiplication and division facts for all numbers up to 12 x 12.
- Use place value, known and derived facts to multiply and divide mentally, including: multiplying and dividing by 10 and 100 and 1.
- Pupils practise to become fluent in the formal written method of short division with exact answers when dividing by a one-digit number
- Pupils practise mental methods and extend this to three-digit numbers to derive facts, for example $200 \times 3 = 600$ so $600 \div 3 = 200$
- Pupils solve two-step problems in contexts, choosing the appropriate operation, working with increasingly harder numbers. This should include correspondence questions such as three cakes shared equally between 10 children.

Year 5 Divide up to 4 digits by a single digit, including those with remainders.

Division – Year 5

Concrete	Pictorial	Abstract
<p data-bbox="190 272 271 296">See Y3</p> <div data-bbox="271 979 750 1422" style="border: 1px solid black; border-radius: 15px; background-color: #4a86e8; color: white; padding: 20px; text-align: center; margin-top: 200px;"> <p data-bbox="324 1209 696 1315">Include money and measure contexts.</p> </div>	<p data-bbox="772 272 853 296">See Y3</p>	<p data-bbox="1355 272 1912 483">Short division with remainders: Now that pupils are introduced to examples that give rise to remainder answers, division needs to have a real life problem solving context, where pupils consider the meaning of the remainder and how to express it, ie. as a fraction, a decimal, or as a rounded number or value , depending upon the context of the problem.</p> <div data-bbox="1355 544 1809 699" style="text-align: center;">  </div> <p data-bbox="1355 767 1890 852">The answer to $5309 \div 8$ could be expressed as 663 and five eighths, $663 \text{ r } 5$, as a decimal, or rounded as appropriate to the problem involved.</p> <div data-bbox="1413 895 1818 1046" style="border: 1px solid black; border-radius: 10px; background-color: #4a86e8; color: white; padding: 10px; text-align: center; margin: 10px auto; width: fit-content;"> <p data-bbox="1440 922 1744 1029">See Y6 for how to continue the short division to give a decimal answer for children who are confident</p> </div> <p data-bbox="1355 1139 1883 1370">If children are confident and accurate: Introduce long division for pupils who are ready to divide any number by a 2-digit number (e.g. $2678 \div 19$). SEE Y6.</p>

Key Vocabulary: share, share equally, one each, two each..., group, equal groups of, lots of, array, divide, divided by, divided into, division, grouping, number line, left, left over, inverse, short division, "carry", remainder, multiple, divisible by, factor, inverse, quotient, prime number, prime factors, composite number (non-prime)

12 ÷ 4 = 3

division sign is equivalent to

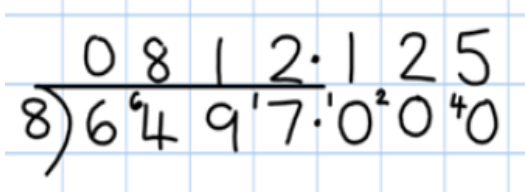
dividend divisor quotient

Division

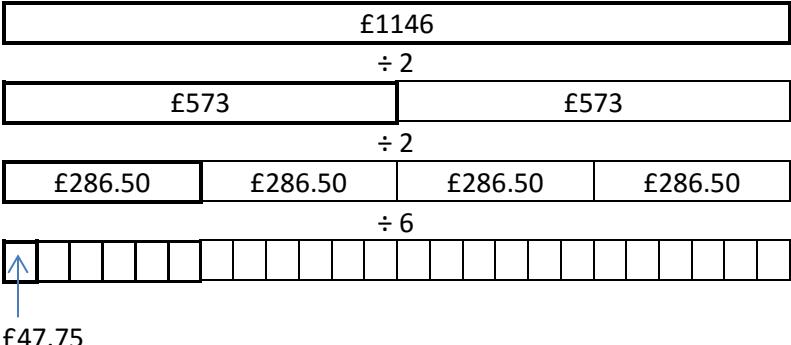
Key number skills needed for division at Y5:

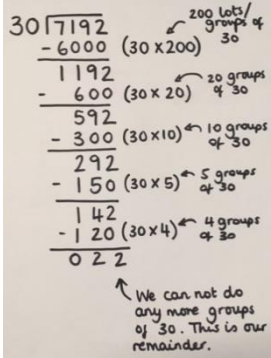
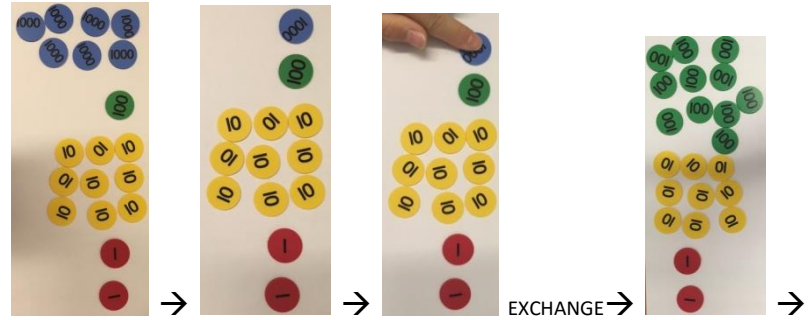
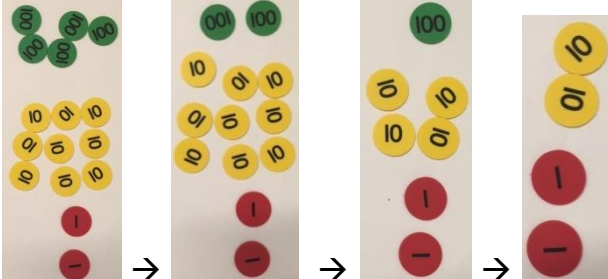
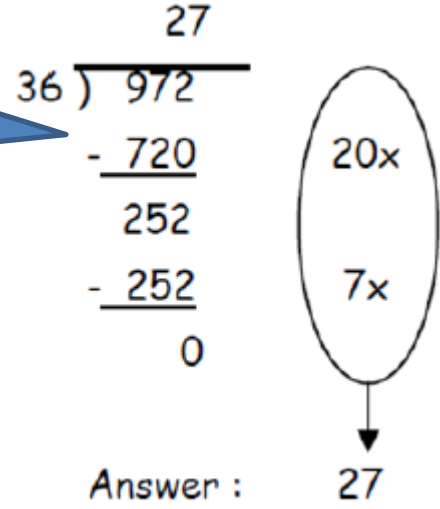
- Recall multiplication and division facts for all numbers up to 12 x 12 (as in Y4).
- Multiply and divide numbers mentally, drawing upon known facts.
- Identify multiples and factors, including finding all factor pairs of a number, and common factors of two number.
- Solve problems involving multiplication and division where larger numbers are decomposed into their factors.
- Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000.
- Use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.
- Work out whether a number up to 100 is prime, and recall prime numbers to 19.
- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- Use multiplication and division as inverses.
- Interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding (e.g. $98 \div 4 = 24 \text{ r } 2 = 24\frac{1}{2} = 24.5 \approx 25$).
- Solve problems involving combinations of all four operations, including understanding of the equals sign, and including division for scaling by different fractions and problems involving simple rates.

Year 6 Divide at least 4 digits by both single-digit and 2-digit numbers (including decimal numbers and quantities)

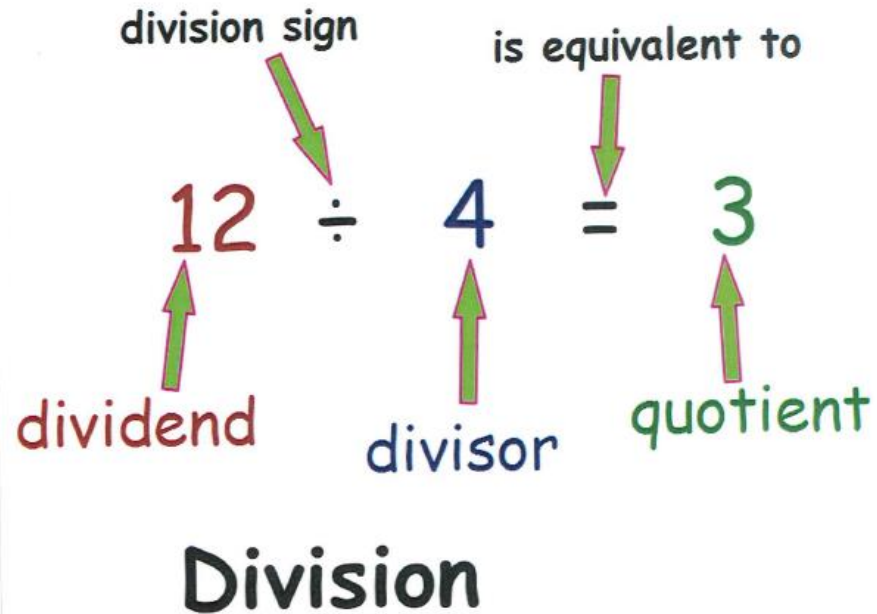
Short division with remainders:		
Concrete	Pictorial	Abstract
See Y3	See Y3	<p>Short division with remainders: Pupils should continue to use this method, but with numbers to at least 4 digits, and understand how to express remainders as fractions, decimals, whole number remainders, or rounded numbers. Real life problem solving contexts need to be the starting point, where pupils have to consider the most appropriate way to express the remainder.</p> <p>Calculating a decimal remainder:</p>  <p>In this example, rather than expressing the remainder as r 1, a decimal point is added after the units because there is still a remainder, and the one remainder is carried onto zeros after the decimal point (to show there was no decimal value in the original number). Keep dividing to an appropriate degree of accuracy for the problem being solved.</p>

Where **remainders** occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.

Short division using FACTORS when dividing by a 2-digit number		
Concrete	Pictorial	Abstract
Use the foam bar models (as pictorial).	<p>Use a bar model</p> <p>The dinner bill came to £1146 for a group of 24 people. They shared the bill equally. How much did each person pay?</p> <p>$£1146 \div 24$</p> <p>$1146 \div 4$ (or $\div 2$ then $\div 2$ again) $\div 6$ IS THE SAME AS $1146 \div 24$</p>  <p>£47.75</p> <p>Explore other pairs of factors that will give the same answer. EG. 3×8 or 12×2 or $3 \times 2 \times 4$ Reasoning: could children prove that this method works with other factor pairs?</p> <p>Link to fractions $\div 24$ is the same as finding $\frac{1}{24}$ of a number.</p>	<p>Children to use their knowledge of factors.</p> <p>Factors can be used to solve division problems:</p> <p>$360 \div 24$ Think of factors of 24. We'll use 6×4 $360 \div 24 = 360 \div 4 \div 6$ $360 \div 4 = 90$ $90 \div 6 = 15$ So $360 \div 24 = 15$</p> <p>$128 \div 20$ Think of factors of 20 \rightarrow 2 and 10 $128 \div 20 = 128 \div 2 \div 10$ $128 \div 2 = 64$ $64 \div 10 = 6.4$</p>

Introducing long division by 'chunking'		
Concrete	Pictorial	Abstract
<p>Link to repeated subtraction. Instead of subtracting one group of the divisor each time, we are subtracting more groups of the divisor each time to find out how many groups of the divisor are in the dividend.</p> <p>Subtract the "chunks" of the divisor, until the answer is zero OR until there is a remainder.</p> <p>Use the PV counters to show the "chunks" being subtracted.</p> <p>EG. $7192 \div 30$</p>    <p>The answer is how many groups have been subtracted in TOTAL. $(200 + 20 + 10 + 5 + 4) = 239 \text{ r } 22$</p>	<p>Children to draw the PV counters. Cross them out when they have been subtracted.</p> <p>Must be aligned in place value for subtracting.</p>	<p>Find out "How many 36s are in 972?" by subtracting "chunks" of 36, until zero is reached (or until there is a remainder). Teach pupils to write a "useful list" first at the side that will help them decide what chunks to use, e.g.:</p> <p>Useful list: $1x = 36$ $10x = 360$ $100x = 3600$</p>  <p>Answer : 27</p> <p>Introduce the method in a simple way by limiting the choice of chunks to "Can we use 10 lots? Can use 100 lots?" As children become confident with the process, encourage more efficient chunks to get to the answer more quickly (e.g. 20x, 5x), and expand on their "useful" lists.</p> <p>Where remainders occur, pupils should express them as fractions, decimals or use rounding, depending upon the problem.</p>

Key Vocabulary: As previously, & common factor



Key number skills needed for division at Y6:

- Recall and use multiplication and division facts for all numbers to 12 x 12 for more complex calculations
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context. Use short division where appropriate.
- Perform mental calculations, including with mixed operations and large numbers.
- Identify common factors, common multiples and prime numbers.
- Solve problems involving all 4 operations.
- Use estimation to check answers to calculations and determine accuracy, in the context of a problem.
- Use written division methods in cases where the answer has up to two decimal places.
- Solve problems which require answers to be rounded to specified degrees of accuracy.

PRESENTATION GUIDELINES

KS1 Maths Presentation Guidelines – Sept '17

Numerical date written in the first row of complete squares.

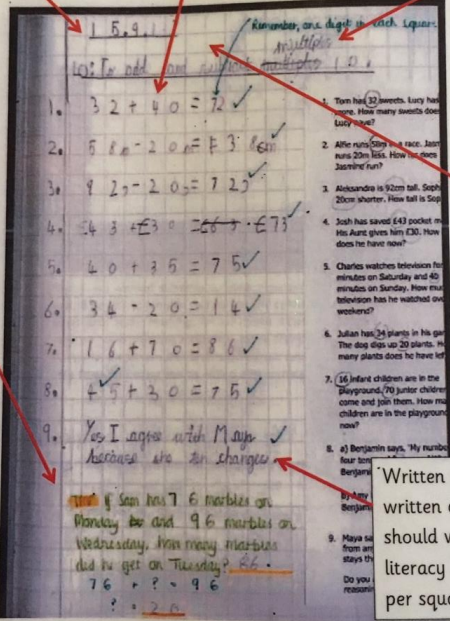
One digit/symbol per square.

Children to write date and short LO (key words from LO or planning)
*Some chn may underline in preparation for KS2.

Draw a margin with a ruler. The margin should be two full squares wide.

Miss a line and write the learning objective (LO: ...)

Written responses should be written on the lines. Children should write like they do in literacy books, **not** one letter per square.



If using a worksheet, the date and LO should already be printed on the worksheet or a space provided for children to write them on.

1.9.09.2017

LO to be written across the page

Date on the right

ONE digit per box

BOTH margins to be two squares wide.

ALL lines drawn with a ruler

Reasoning questions to be stuck in one at a time.

Task A:
True or false?
Are these number sentences true or false?
 $6.17 + 0.4 = 6.57$
 $8.12 - 0.9 = 8.3$
Give your reasons.

$$\begin{array}{r} 6.17 \\ + 0.4 \\ \hline 6.57 \end{array}$$

= 6.57 True

I used partitioning to help me calculate mentally.

Decimal point on the line (not an individual box) and in the middle 0.4

$$\begin{array}{r} 0.4 \\ \times \\ 0.4 \\ \hline \end{array}$$

Encourage children to give detailed written responses to explain their reasoning.

$8.12 - 0.9$ is NOT 8.3 . 0.9 is nearly 1. If I take away 1, I get 7.12 and 8.3 is not close to this. 8.3 is actually bigger than the number that I originally started with.